Chapter 7

PERT and PERT simulation

PERT—PROGRAM EVALUATION AND REVIEW TECHNIQUE

In 1958 the Special Office of the Navy and the Booze, Allen and Hamilton consulting firm developed PERT (program evaluation and review technique) to schedule the more than 3300 contractors of the Polaris submarine project and to cover uncertainty of activity time estimates.

PERT is almost identical to the critical path method (CPM) technique except it assumes each activity duration has a range that follows a statistical distribution. PERT uses three time estimates for each activity. Basically, this means each activity duration can range from an optimistic time to a pessimistic time, and a weighted average can be computed for each activity. Because project activities usually represent work, and because work tends to stay behind once it gets behind, the PERT developers chose an approximation of the *beta distribution* to represent activity durations. This distribution is known to be flexible and can accommodate empirical data that do not follow a normal distribution. The activity durations can be skewed more toward the high or low end of the data range. Figure A14.1A depicts a *beta distribution* for activity durations that is skewed toward the right and is representative of work that tends to stay late once it is behind. The distribution for the project duration is represented by a normal (symmetrical) distribution shown in Figure A14.1B. The project distribution represents the sum of the weighted averages of the activities on the critical path(s).

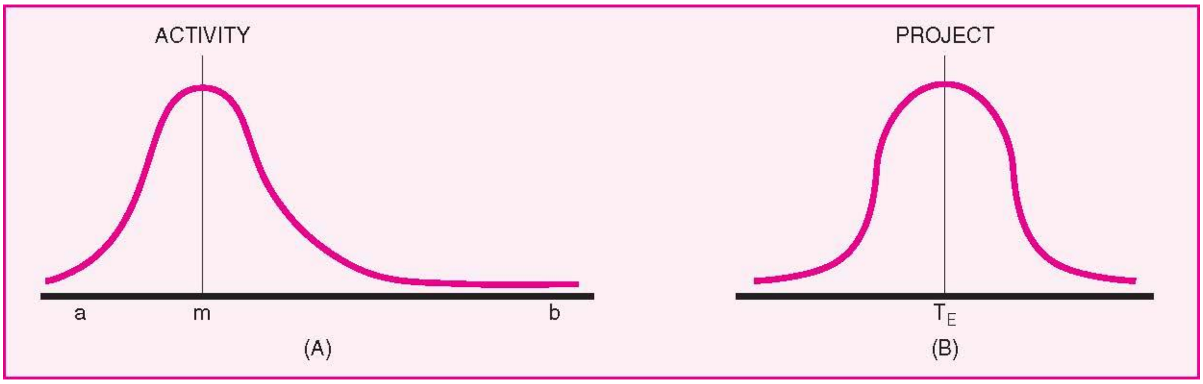

FIGURE A14.1 Activity and project frequency distributions

Knowing the weighted average and variances for each activity allows the project planner to compute the probability of meeting different project durations. Follow the steps described in the hypothetical example given next. (The jargon is difficult for those not familiar with statistics, but the process is relatively simple after working through a couple of examples.)

The weighted average activity time is computed by the following formula:

$$t_e = \frac{a + 4m + b}{6} \qquad (14.1)$$

where:

t_e = weighted average activity time

a = optimistic activity time (1 chance in 100 of completing the activity earlier under *normal* conditions)

b = pessimistic activity time (1 chance in 100 of completing the activity later under *normal* conditions)

m = most likely activity time.

When the three time estimates have been specified, this equation is used to compute the weighted average duration for each activity. The average (deterministic) value is placed on the project network as in the CPM method and the early, late, slack and project completion times are computed as they are in the CPM method.

The variability in the activity time estimates is approximated by the following equations: Equation 14.2 represents the standard deviation for the *activity.* Equation 14.3 represents the standard deviation for the *project.* Note the standard deviation of the activity is squared in this equation; this is also called variance. This sum includes only activities on the critical path(s) or path being reviewed.

$$\sigma_{t_e} = \left(\frac{b-a}{6}\right) \qquad (14.2)$$

$$\sigma_{T_E} = \sqrt{\Sigma\sigma_{t_e}^2} \qquad (14.3)$$

Finally, the average project duration (T_E) is the sum of all the average activity times along the critical path (sum of t_e), and it follows a normal distribution.

Knowing the average project duration and the variances of activities allows the probability of completing the project (or segment of the project) by a specific time to be computed using standard statistical tables. The equation below (Equation 14.4) is used to compute the *'Z'* value found in statistical tables (Z = number of standard deviations from the mean), which, in turn, tells the probability of completing the project in the time specified.

$$Z = \frac{T_S - T_E}{\sqrt{\Sigma\sigma_{t_e}^2}} \qquad (14.4)$$

where:

T_E = critical path duration

T_S = scheduled project duration

Z = probability (of meeting scheduled duration) found in statistical Table A14.2

A hypothetical example using the PERT technique

The activity times and variances are given in Table A14.1. The project network is presented in Figure A14.2. This figure shows the project network as AON.

TABLE A14.1 Activity times and variances

Activity	a	m	b	t_e	$[(b-a)/6]^2$
1–2	17	29	47	30	25
2–3	6	12	24	13	9
2–4	16	19	28	20	4
3–5	13	16	19	16	1
4–5	2	5	14	6	4
5–6	2	5	8	5	1

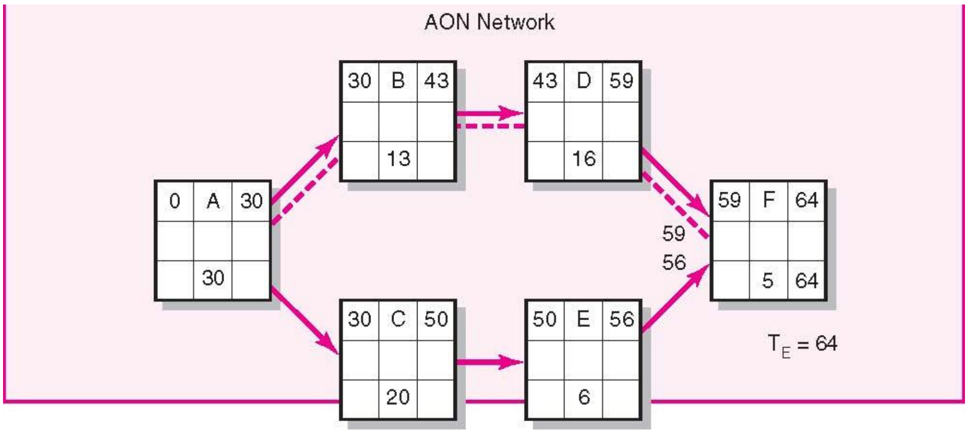

FIGURE A14.2 Hypothetical network

The expected project duration (T_E) is 64 time units; the critical path is 1-2-3-5-6. With this information, the probability of completing the project by a specific date can easily be computed using standard statistical methods. For example, what is the probability the project will be completed before a scheduled time (T_S) of 67? The normal curve for the project would appear as shown in Figure A14.3.

Using the formula for the *Z* value, the probability can be computed as follows:

$$Z = \frac{T_S - T_E}{\sqrt{\Sigma\sigma_{t_e}^2}}$$

$$= \frac{67 - 64}{\sqrt{25 + 9 + 1 + 1}}$$

$$= \frac{+3}{\sqrt{36}}$$

$$= +0.50$$

$$P = 0.69$$

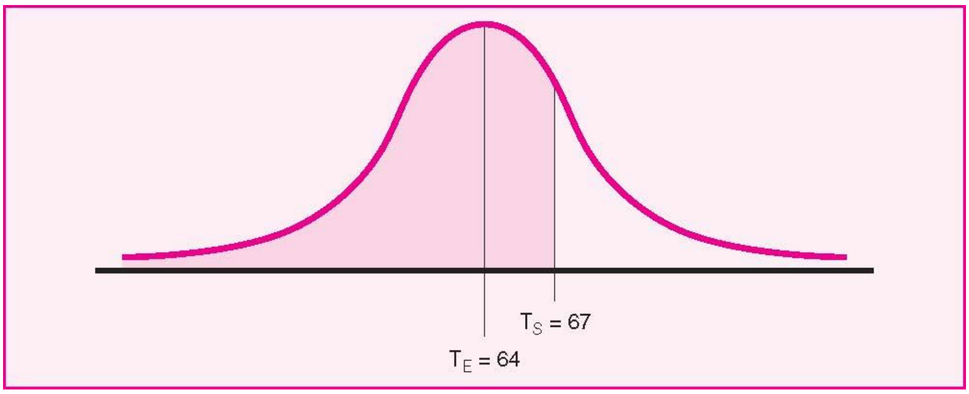

FIGURE A14.3 Possible project durations

Reading from Table A14.2, a *Z* value of +0.5 gives a probability of 0.69, which is interpreted to mean there is a 69 per cent chance of completing the project on or before 67 time units.

Conversely, the probability of completing the project by time period 60 is computed as follows:

$$Z = \frac{60 - 64}{\sqrt{25 + 9 + 1 + 1}}$$
$$= \frac{-4}{\sqrt{36}}$$
$$= -0.67$$
$$P \approx 0.26$$

From Table A14.2, a *Z* value of –0.67 gives an approximate probability of 0.26, which is interpreted to mean there is about a 26 per cent chance of completing the project on or before 60 time units. Note that this same type of calculation can be made for any path or segment of a path in the network.

Table A14.2 Z Values and probabilities

Z Value	Probability	*Z* Value	Probability
−3.0	.001	+0.0	.500
−2.8	.003	+0.2	.579
−2.6	.005	+0.4	.655
−2.4	.008	+0.6	.726
−2.2	.014	+0.8	.788
−2.0	.023	+1.0	.841
−1.8	.036	+1.2	.885
−1.6	.055	+1.4	.919
−1.4	.081	+1.6	.945
−1.2	.115	+1.8	.964
−1.0	.159	+2.0	.977
−0.8	.212	+2.2	.986
−0.6	.274	+2.4	.992
−0.4	.345	+2.6	.995
−0.2	.421	+2.8	.997

When such probabilities are available to management, trade-off decisions can be made to accept or reduce the risk associated with a particular project duration. For example, if the project manager wishes to improve the chances of completing the project by 64 time units, at least two choices are available. First, management can spend money up front to change conditions that will reduce the duration of one or more activities on the critical path. A more prudent, second alternative would be to allocate money to a contingency fund and wait to see how the project is progressing as it is implemented.

EXERCISES

1. Given the project information below, what is the probability of completing the National Holiday Toy project in 93 time units?

Act. ID	Description	Predecessor	Optm. (a)	Most likely (m)	Pess. (b)	Act time t_e	Variance $[(b\ 2\ a)/6]^2$	Critical
1	Design package	None	6	12	24			
2	Design product	1	16	19	28			
3	Build package	1	4	7	10			
4	Secure patent	2	21	30	39			
5	Build product	2	17	29	47			
6	Paint	3, 4, 5	4	7	10			
7	Test market	6	13	16	19			

2. The Global Tea and Organic Juice companies have merged.
The following information has been collected for the 'Consolidation Project.'

Activity	Description	Predecessor	a opt	m ml	b pess
1	Codify accounts	None	16	19	28
2	File articles of unification	None	30	30	30
3	Unify price and credit policy	None	60	72	90
4	Unify personnel policies	None	18	27	30
5	Unify data processing	1	17	29	47
6	Train accounting staff	1	4	7	10
7	Pilot run data processing	5	12	15	18
8	Calculate P&L and balance sheet	6, 7	6	12	24
9	Transfer real property	2	18	27	30
10	Train sales force	3	20	35	50
11	Negotiate with unions	4	40	55	100
12	Determine capital needs	8	11	20	29
13	Explain personnel policies	11	14	23	26
14	Secure line of credit	9, 12	13	16	19
15	End	10, 12, 14	0	0	0

a) Compute the expected time for each activity.
b) Compute the variance for each activity.
c) Compute the expected project duration.
d) What is the probability of completing the project by day 112? Within 116 days?
e) What is the probability of completing 'Negotiate with Unions' by day 90?

3. **The expected times and variances for the project activities are given below. What is the probability of completing the project in 25 periods?**

Variance ID	**Description**	**Predecessor**	t_e	$[(b\ 2\ a)/6]^2$
1	Pilot production	None	6	3
2	Select channels of distrib.	None	7	4
3	Develop mktg. program	None	4	2
4	Test market	1	4	2
5	Patent	1	10	5
6	Full production	4	16	10
7	Ad promotion	3	3	2
8	Release	2, 5, 6, 7	2	1