# Homework Chapter 9

**1.** A population is normally distributed with $\mu$ = 100 and $\sigma$= 20.
a. Find the probability that a value randomly selected from this population will have a value greater than 130.
b. Find the probability that a value randomly selected from this population will have a value less than 90.
c. Find the probability that a value randomly selected from this population will have a value between 90 and 130.

**2.** Suppose that a population is known to be normally distributed with $\mu$ = 2,000 and $\sigma$ = 230. If a random sample of size $n$ = 8 is selected, calculate the probability that the sample mean will exceed 2,100.

**3.** Assuming the population of interest is approximately normally distributed, construct a 95% confidence interval estimate for the population mean given the following values: $\bar{x}$= 18.4 $s$ = 4.2 $n$ = 13

**4.** Construct a 95% confidence interval estimate for the population mean given the following values: $\bar{x}$= 300 $\sigma$ = 55 $n$ = 250

**5.** An advertising company wishes to estimate the mean household income for all single working professionals who own a foreign automobile. If the advertising company wants a 90% confidence interval estimate with a margin of error of ±$2,500, what sample size is needed if the population standard deviation is known to be $27,500?

**Answers**

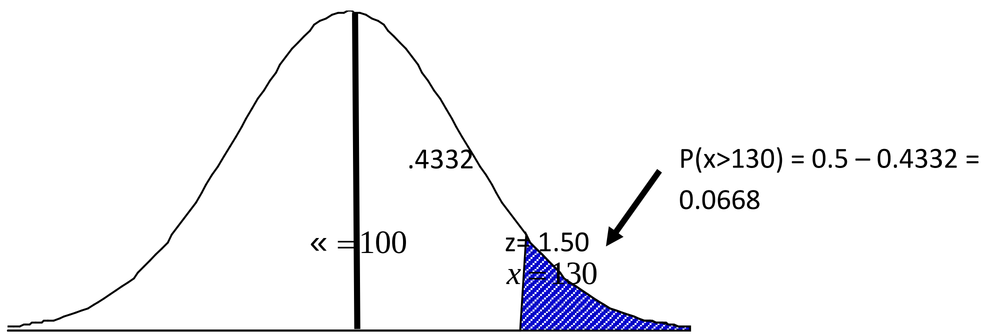
1.

a. We are given the mean and standard deviation as follows: $« = 100$ and

The event of interest is $P(x > 130) = ?$

We find the z-value using: $z = \frac{x - «}{±} = \frac{130 - 100}{20} = \frac{30}{20} = 1.50$

The probability from the standard normal table associated with z = 1.50 is 0.4332. This corresponds to the area between z = 1.50 and the mean. To desired probability is found by subtracting 0.4332 from 0.5000 giving 0.0668. This is illustrated in the following graph.

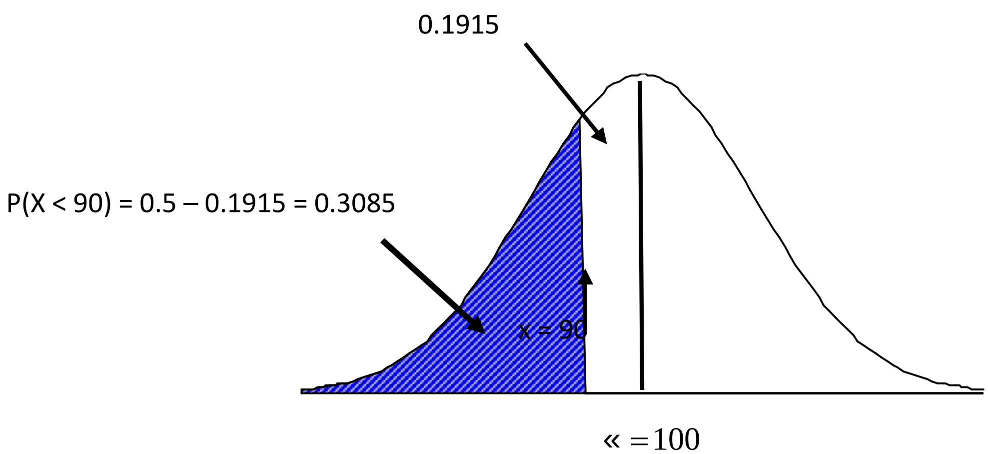



b. We are given the mean and standard deviation as follows: $« = 100$ and

The event of interest is $P(x < 90) = ?$

We find the z-value using: $z = \frac{x - «}{±} = \frac{90 - 100}{20} = \frac{-10}{20} = -0.50$

The probability from the standard normal table associated with z = -.50 is 0.1915. This corresponds to the area between z = -.50 and the mean. To desired probability is found by subtracting 0.1915 from 0.5000 giving 0.3085. This is illustrated in the following graph.




c. We are given the mean and standard deviation as follows: $« = 100$ and

The event of interest is $P(90 \le x \le 130) = ?$

We find the z-value using: $z = \frac{x - «}{\pm} = \frac{90 - 100}{20} = \frac{-10}{20} = -0.50$

and $z = \frac{x - «}{\pm} = \frac{130 - 100}{20} = \frac{30}{20} = 1.50$

The probability from the standard normal table associated with z = -.50 is 0.1915. The probability associated with z = 1.50 is .4332. These corresponds to the areas between z = -.50 and the mean and z = 1.50 and the mean. To desired probability is found by adding the two probabilities:

$$P(90 \le x \le 130) = 0.1915 + 0.4332 = 0.6247$$

2. Because the population is normally distributed, the sampling distribution for the mean will also be normally distributed. Thus, the following steps can be used to answer this question:

Step 1: Determine the sample mean.

The sample mean is given to be $\bar{x} = 2{,}100$

Step 2: Define the sampling distribution.

The sampling distribution will be normally distributed and will have $«_{\bar{x}} = « = 2000$ and a standard deviation equal to $\pm_{\bar{x}} = \frac{\pm}{\sqrt{n}} = \frac{230}{\sqrt{8}} = 81.32$

Step 3: Define the event of interest.

We are interested in the following:

$$P(\bar{x} > 2{,}100) = ?$$

Step 4: Convert the sample mean to a standardized z value.

$$z = \frac{\bar{x} - «}{\frac{\pm}{\sqrt{n}}} = \frac{2{,}100 - 2{,}000}{\frac{230}{\sqrt{8}}} = \frac{100}{81.32} = 1.23$$

Step 5: Use the standard normal distribution to find the desired probability.

The probability associated with a z-value of 1.23 from the standard normal table is 0.3907. Then

$$P(\bar{x} > 2{,}100) = 0.5000 - 0.3907 = 0.1093$$

Thus, there is slightly more than a 0.10 chance that a sample mean exceeding 2,100 would come from this population if a sample size of n = 8 is selected.

3. Since the population standard deviation is unknown, the following steps can be used to compute the confidence interval estimate.

Step 1: Define the population of interest and select a simple random sample.

The population of interest is the collection of all items of interest. A simple random sample of size n = 13 will be collected.

Step 2: Specify the confidence level.

The desired confidence level is 95%.

Step 3: Compute the sample mean and the sample standard deviation.

The sample mean and sample standard deviation are given to be:

$$\bar{x} = 18.4$$

Step 4: Determine the standard error of the sampling distribution.

The sample standard deviation is computed to be $s = 4.2$. The standard error of the sampling distribution is:

$$\pm_{\bar{x}} \approx \frac{s}{\sqrt{n}} = \frac{4.2}{\sqrt{13}} = 1.16$$

Step 5: Determine the critical value for the desired confidence level.

The critical value for 95% confidence from the student t-distribution table with degrees of freedom equal to n – 1 = 12 is *t* = 2.1788.

Step 6: Compute the confidence interval estimate.

The 95% confidence interval estimate for the population mean is:

$$\bar{x} \pm t \frac{s}{\sqrt{n}}$$

Therefore the confidence interval is:

$$18.4 \pm 2.1788 \frac{4.2}{\sqrt{13}}$$

$$18.4 \pm 2.54$$

15.86 -------------------------------- 20.94

4. Since the population standard deviation is known, the following steps can be used to develop the desired confidence interval estimate.

Step 1: Define the population of interest and select a simple random sample.

The population of interest is the collection of all items of interest. A simple random sample of size n = 250 will be collected.

Step 2: Specify the confidence level.

The desired confidence level is 95%.

Step 3: Compute the sample mean.

The sample mean is given to be $\bar{x} = 300$.

Step 4: Determine the standard error of the sampling distribution.

The population standard deviation is known to be $\pm = 55$. The standard error of the sampling distribution is:

$$\pm_{\bar{x}} = \frac{\pm}{\sqrt{n}} = \frac{55}{\sqrt{250}} = 3.48$$

Step 5: Determine the critical value, z, from the standard normal distribution table.

The z value is 1.96.

Step 6: Compute the confidence interval estimate.

The 95% confidence interval estimate for the population mean is:

$$\bar{x} \pm z \frac{\pm}{\sqrt{n}}$$

The critical value for 95% confidence from the standard normal distribution table is z = 1.96.
Therefore the confidence interval is:

$$300 \pm 1.96 \frac{55}{\sqrt{250}}$$

$$300 \pm 6.82$$

293.18 -------------------------------- 306.82

5.

The required sample size is n = $\frac{z^2 \pm^2}{e^2} = \frac{1.645^2 27500^2}{2500^2} = 327.43 = 328$