QM353: Business Statistics

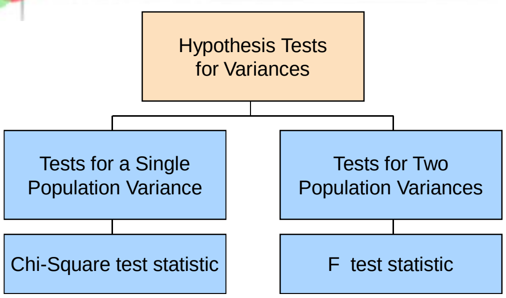
Chapter 4
Hypothesis Tests and Estimation for Population Variances

Chapter Goals

After completing this chapter, you should be able to:

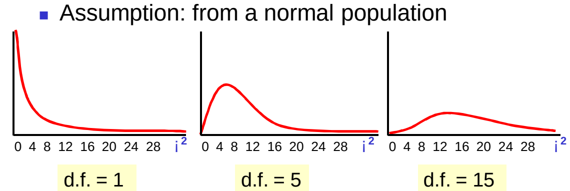
- Formulate and complete hypothesis tests for a single population variance
- Find critical chi-square distribution values from the chi-square table
- Formulate and complete hypothesis tests for the difference between two population variances

Hypothesis Tests for Variances



The Chi-Square Distribution

- The chi-square distribution is a family of distributions, depending on degrees of freedom:
- d.f. = n – 1
- Assumption: from a normal population



Chi-Square Test Statistic

The chi-squared test statistic for a Single Population Variance is:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

where
χ^2 = standardized chi-square variable
n = sample size
s^2 = sample variance
σ^2 = hypothesized variance

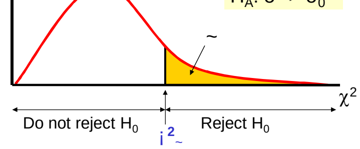
The test statistic standardizes the sample variance similar to the use of z and t from previous chapters

Finding the Critical Value

- The critical value, ¡ 2 ~ , is found from the chi-square table (Appendix G)

Upper tail test:

H_0: $\sigma^2 \le \sigma_0^2$
H_A: $\sigma^2 > \sigma_0^2$

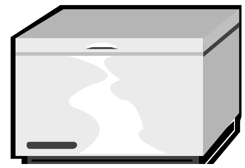


Hypothesis Test with Chi-Square

1. Formulate the hypotheses in terms of σ^2
2. Specify the level of significance
3. Construct the rejection region
4. Compute the test statistic, χ^2
5. Reach a decision
6. Draw a conclusion

Example

- A commercial freezer must hold the selected temperature with little variation. Specifications call for a standard deviation of no more than 4 degrees (or variance of 16 degrees^2).
- A sample of 16 freezers is tested and yields a sample variance of $s^2 = 24$. Test to see whether the standard deviation specification is exceeded. Use ~ = 0.05



Finding the Critical Value

- Use the chi-square table to find the critical value:

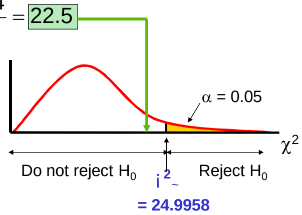
  $i^2_{\sim}$ = 24.9958 (~ = 0.05 and 16 – 1 = 15 d.f.)

The test statistic is:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(16-1)24}{16} = 22.5$$

Since 22.5 < 24.9958, do not reject H_0

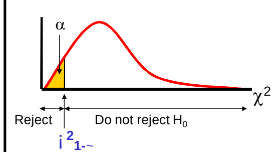
There is not significant evidence at the ~ = 0.05 level that the standard deviation specification is exceeded



Lower Tail or Two Tailed Chi-square Tests

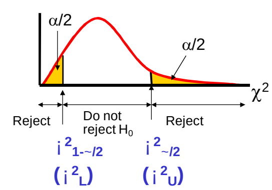
Lower tail test:

H_0: $\sigma^2 \ge \sigma_0^2$
H_A: $\sigma^2 < \sigma_0^2$



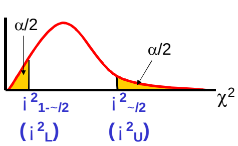
Two tail test:

H_0: $\sigma^2 = \sigma_0^2$
H_A: $\sigma^2 \ne \sigma_0^2$



Confidence Interval Estimate for σ^2

- The confidence interval estimate for σ^2 is

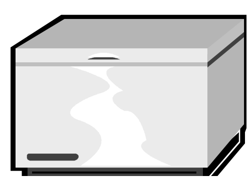


$$\frac{(n-1)s^2}{\chi^2_U} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_L}$$

Where χ^2_L and χ^2_U are from the χ^2 distribution with n -1 degrees of freedom

Example

- A sample of 16 freezers yields a sample variance of $s^2 = 24$.
- Form a 95% confidence interval for the population variance.

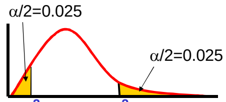


Example

(continued)

- Use the chi-square table to find χ^2_L and χ^2_U :

  (~ = 0.05 and 16 – 1 = 15 d.f.)



$\chi^2_{0.975}$ (χ^2_L)	$\chi^2_{0.025}$ (χ^2_U)
6.2621	27.4884

$$\frac{(n-1)s^2}{\chi^2_U} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_L}$$

$$\frac{(16-1)24}{27.4884} \le \sigma^2 \le \frac{(16-1)24}{6.2621}$$

$$13.096 \le \sigma^2 \le 57.489$$

We are 95% confident that the population variance is between 13.096 and 57.489 degrees². (Taking the square root, we are 95% confident that the population standard deviation is between 3.619 and 7.582 degrees.)

The F Distribution

- The F critical value is found from the F table
- The are two appropriate degrees of freedom:

  D_1 (numerator) and D_2 (denominator)
- Assumes the populations follow the normal distribution
- Assumes the samples are randomly selected and independent of each other
- Is the ratio of 2 independent chi-square distributions
- In the F table (Appendix H),
  - numerator degrees of freedom determine the row
  - denominator degrees of freedom determine the column

F Test for Difference in Two Population Variances

The F test statistic is:

$$F = \frac{s_1^2}{s_2^2}$$

Where F has D_1 numerator and D_2 denominator degrees of freedom

s_1^2 = Variance of Sample 1

$D_1 = n_1 - 1$ = numerator degrees of freedom

s_2^2 = Variance of Sample 2

$D_2 = n_2 - 1$ = denominator degrees of freedom

Formulating the F Ratio

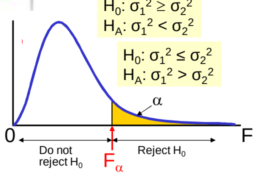
$$F = \frac{s_1^2}{s_2^2}$$ where $D_1 = n_1 - 1$; $D_2 = n_2 - 1$

- For a two-tailed test, always place the larger sample variance in the numerator
- For a one-tailed test, consider the alternative hypothesis: place in the numerator the sample variance for the population that is predicted (based on H_A) to have the larger variance

Finding the Critical Value

$H_0: \sigma_1^2 \ge \sigma_2^2$
$H_A: \sigma_1^2 < \sigma_2^2$

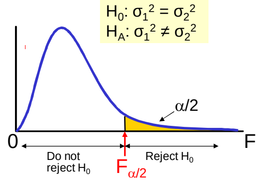
$H_0: \sigma_1^2 \le \sigma_2^2$
$H_A: \sigma_1^2 > \sigma_2^2$



- rejection region for a one-tail test is

$$F = \frac{s_1^2}{s_2^2} > F_\alpha$$

$H_0: \sigma_1^2 = \sigma_2^2$
$H_A: \sigma_1^2 \ne \sigma_2^2$



- rejection region for a two-tailed test is

$$F = \frac{s_1^2}{s_2^2} > F_{\alpha/2}$$

(where the larger sample variance in the numerator)

Finding the Critical Value

(continued)

- For a two-tailed test use the table corresponding to α/2
  - e.g., if α = 0.10, use the F table with the upper tail equal to 0.05
- For a one-tailed test, use the F table corresponding to the actual significance level
  - e.g., if α = 0.05, use the F table with the upper tail equal to 0.05

F Test: An Example

You are a financial analyst for a brokerage firm. You want to compare dividend yields between stocks listed on the NYSE & NASDAQ. You collect the following data:

	NYSE	NASDAQ
Number	21	25
Mean	3.27	2.53
Std dev	1.30	1.16

Is there a difference in the variances between the NYSE & NASDAQ at the ~ = 0.05 level?

F Test: Example Solution

- Form the hypothesis test:

  $H_0: \sigma^2_1 = \sigma^2_2$ (there is no difference between variances)

  $H_A: \sigma^2_1 \neq \sigma^2_2$ (there is a difference between variances)

- Find the F critical value for ~ = 0.05:
  - Numerator:
    - $D_1 = n_1 - 1 = 21 - 1 = 20$
  - Denominator:
    - $D_2 = n_2 - 1 = 25 - 1 = 24$

  $F_{0.05/2,\,20,\,24} = 2.327$

NOTE: Make sure the sample with the largest variance corresponds to D_1

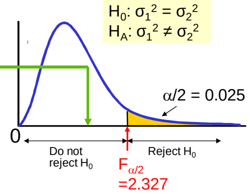
F Test: Example Solution

(continued)

- The test statistic is:

$$F = \frac{s_1^2}{s_2^2} = \frac{1.30^2}{1.16^2} = 1.256$$

$H_0: \sigma_1^2 = \sigma_2^2$
$H_A: \sigma_1^2 \neq \sigma_2^2$



- F = 1.256 is not greater than the critical F value of 2.327, so we do not reject H_0
- Conclusion: There is no evidence of a difference in variances at ~ = 0.05

Chapter Summary

- Performed chi-square tests for the variance for a single population
  - Used the chi-square table to find chi-square critical values
- Performed F tests for the difference between two population variances
  - Used the F table to find F critical values