QM353: Business Statistics

Chapter 3
Hypothesis Testing on Population Parameters

Chapter Goals

After completing this chapter, you should be able to:

- Formulate null and alternative hypotheses involving a single population mean or proportion
- Know what Type I and Type II errors are
- Formulate a decision rule for testing a hypothesis
- Know how to use the test statistic, critical value, and p-value approaches to test the null hypothesis

Chapter Goals

After completing this chapter, you should be able to:

- Understand the logic of hypothesis testing
- Test hypotheses and form interval estimates
  - Two independent population means
    - Standard deviations known
    - Standard deviations unknown
  - The difference between two population proportions

What is Hypothesis Testing?

- Statistical inference
  - Estimating population parameters based on sample statistics
- An analytical method for making decisions
- Through gathering statistical evidence a claim about a population can be accepted or rejected
  - Must have enough evidence to reject, otherwise you accept the claim
- A procedure that incorporates sampling error
  - We never actually 100% “prove” anything because of sampling error

What is a Hypothesis?

- A hypothesis is a claim (assumption) about a population parameter:

  - population mean

Example: The mean monthly cell phone bill of this city is $\mu = \$42$

  - population proportion

Example: The proportion of adults in this city with cell phones is $\pi = 0.68$

The Null Hypothesis, H_0

- States the assumption or default position (numerical) to be tested

Example: The average number of TV sets in U.S. Homes is at least three ($H_0: \mu \ge 3$)

- Is always about a population parameter, not about a sample statistic

$H_0: \mu \ge 3$ $H_0: \bar{x} \ge 3$

The Null Hypothesis, H_0

(continued)

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty
- Refers to the status quo
- Always contains "=" , "≤" or "ñ" sign
- May or may not be rejected
  - Based on the statistical evidence gathered

The Alternative Hypothesis, H_A

- Is the opposite of the null hypothesis
  - e.g.: The average number of TV sets in U.S. homes is less than 3 ($H_A: \mu < 3$)
- Challenges the status quo
- Never contains the "=" , "≤" or "ñ" sign
- May or may not be accepted
- Is generally the hypothesis that is believed (or needs to be supported) by the researcher – a research hypothesis

Formulating Hypotheses

- Example 1: Ford Motor Company has worked to reduce road noise inside the cab of the redesigned F150 pickup truck. It would like to report in its advertising that the truck is quieter. The average of the prior design was 68 decibels at 60 mph.

What is the appropriate hypothesis test?

Formulating Hypotheses

- Example 1: Ford Motor Company has worked to reduce road noise inside the cab of the redesigned F150 pickup truck. It would like to report in its advertising that the truck is quieter. The average of the prior design was 68 decibels at 60 mph.

- What is the appropriate test?

$H_0: \mu \geq 68$ (the truck is not quieter) status quo
$H_A: \mu < 68$ (the truck is quieter) wants to support

- If the null hypothesis is rejected, Ford has sufficient evidence to support that the truck is now quieter.

Formulating Hypotheses

- Example 2: The average annual income of buyers of Ford F150 pickup trucks is claimed to be $65,000 per year. An industry analyst would like to test this claim.

What is the appropriate hypothesis test?

Formulating Hypotheses

- Example 1: The average annual income of buyers of Ford F150 pickup trucks is claimed to be $65,000 per year. An industry analyst would like to test this claim.

- What is the appropriate test?

$H_0: \mu = 65{,}000$ (income is as claimed) status quo
$H_A: \mu \neq 65{,}000$ (income is different than claimed)

- The analyst will believe the claim unless sufficient evidence is found to discredit it.

Errors in Making Decisions

3 outcomes for a hypothesis test

1. No error
2. Type I error
3. Type II error

Errors in Making Decisions

(continued)

- **Type I Error**
  - Rejecting a true null hypothesis
  - Considered a serious type of error

The probability of Type I Error is ~

- Called level of significance of the test
- Set by researcher in advance

Errors in Making Decisions

(continued)

- **Type II Error**
  - Failing to reject (i.e., accept) a false null hypothesis

The probability of Type II Error is β

- β is a calculated value

Hypothesis Testing Process

Claim: **the population mean age is 50.**

Null Hypothesis: H_0: $\mu = 50$

Population

Now select a random sample:

Sample

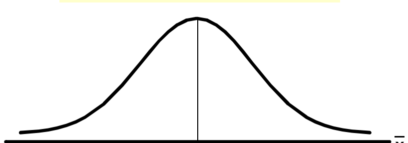
Suppose the sample mean age is 20: $\bar{x} = 20$

Is $\bar{x} = 20$ likely if $\mu = 50$?

If not likely, REJECT Null Hypothesis

Reason for Rejecting H_0

Sampling Distribution of $\bar{x}$



20 — If it is unlikely that we would get a sample mean of this value ...

$\mu = 50$ If H_0 is true — ... if in fact this were the population mean…

... then we reject the null hypothesis that $\mu = 50$

Outcomes and Probabilities

Possible Hypothesis Test Outcomes

	State of Nature	
Decision	H_0 True	H_0 False
Do Not Reject H_0	**No error** **(1 - a)**	**Type II Error** **(β)**
Reject H_0	**Type I Error** **(a)**	**No Error** **(1 - β)**

Key: **Outcome** **(Probability)**

Type I & II Error Relationship

- Type I and Type II errors cannot happen at the same time
  - Type I error can only occur if H_0 is true
  - Type II error can only occur if H_0 is false

> If Type I error probability (α) ⬇, then
> Type II error probability (β) ⬆

Level of Significance, ~

- **Defines unlikely values of sample statistic if null hypothesis is true**
  - Defines rejection region of the sampling distribution
- Is designated by ~ , (level of significance)
  - Typical values are 0.01, 0.05, or 0.10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

Hypothesis Tests for the Mean

Hypothesis Tests for «

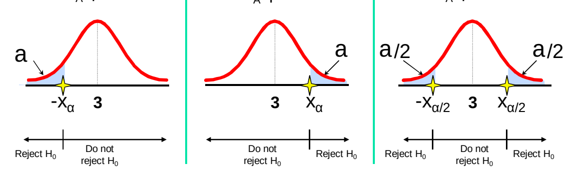
- **σ Known**
- **σ Unknown**

- Assume first that the population standard deviation σ is known

Level of Significance and the Rejection Region

Level of significance = a

Lower tail test	Upper tail test	Two tailed test
Example:	Example:	Example:
H_0: μ ≥ 3	H_0: μ ≤ 3	H_0: μ = 3
H_A: μ < 3	H_A: μ > 3	H_A: μ ≠ 3

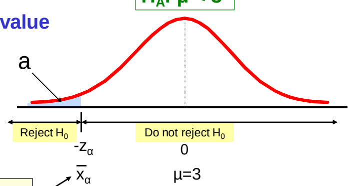


Critical Value for Lower Tail Test

H_0: μ ≥ 3
H_A: μ < 3

- The cutoff value, $-z_\alpha$ or $\bar{x}_\alpha$, is called a **critical value**

based on a

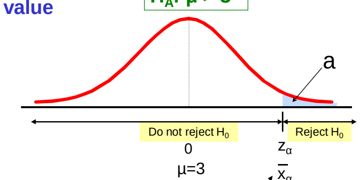


$$\bar{x}_\alpha = \mu - z_\alpha \frac{\sigma}{\sqrt{n}}$$

Critical Value for Upper Tail Test

H_0: μ ≤ 3
H_A: μ > 3

- The cutoff value, z_α or $\bar{x}_\alpha$, is called a **critical value**



$$\bar{x}_\alpha = \mu + z_\alpha \frac{\sigma}{\sqrt{n}}$$

Critical Values for Two Tailed Tests

- There are two cutoff values (critical values):

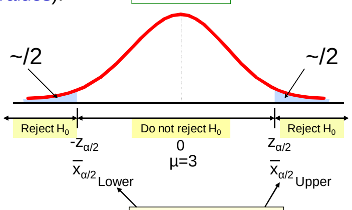
$\pm z_{\alpha/2}$

or

$\bar{x}_{\alpha/2}$ Lower

$\bar{x}_{\alpha/2}$ Upper

H_0: μ = 3
H_A: μ ÷ 3



$$\bar{x}_{\alpha/2} = \mu \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Two Equivalent Approaches to Hypothesis Testing

- z-units:
  - For given α, find the critical z value(s):
    - $-z_\alpha$, z_α ,or $\pm z_{\alpha/2}$
  - Convert the sample mean $\bar{x}$ to a z test statistic: $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$
  - Reject H_0 if z is in the rejection region, otherwise do not reject H_0
- $\bar{x}$ units:
  - Given α, calculate the critical value(s)
    - $\bar{x}_\alpha$, or $\bar{x}_{\alpha/2(L)}$ and $\bar{x}_{\alpha/2(U)}$
  - The sample mean is the test statistic. Reject H_0 if $\bar{x}$ is in the rejection region, otherwise do not reject H_0

Process of Hypothesis Testing

1. Specify population parameter of interest
2. Formulate the null and alternative hypotheses
3. Specify the desired significance level, α
4. Define the rejection region
5. Take a random sample and determine whether or not the sample result is in the rejection region
6. Reach a decision and draw a conclusion

Hypothesis Testing Example

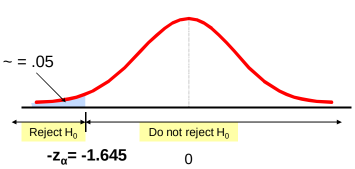
Test the claim that the true mean # of TV sets in US homes is at least 3. (Assume σ = 0.8)

1. Specify the population value of interest
   - The mean number of TVs in US homes
2. Formulate the appropriate null and alternative hypotheses
   - H_0: μ ≥ 3 H_A: μ < 3 (This is a lower tail test)
3. Specify the desired level of significance
   - Suppose that ~ = 0.05 is chosen for this test

Hypothesis Testing Example

(continued)

- 4. Determine the rejection region



This is a one-tailed test with ~ = 0.05.
Since **σ is known**, the cutoff value is a z value:

Reject H_0 if $z < z_\alpha = -1.645$; otherwise do not reject H_0

Hypothesis Testing Example

- 5. Obtain sample evidence and compute the test statistic

Suppose a sample is taken with the following results: n = 100, $\bar{x}$ = 2.84 (σ = 0.8 is assumed known)

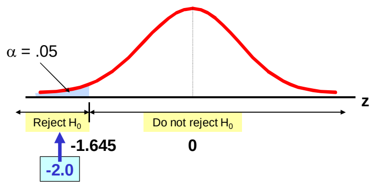
- Then the test statistic is:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{0.08} = -2.0$$

Hypothesis Testing Example

(continued)

- 6. Reach a decision and interpret the result

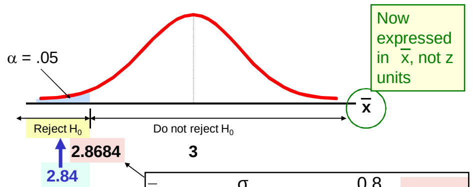


Since $z = -2.0 < -1.645$, we reject the null hypothesis that the mean number of TVs in US homes is at least 3. There is sufficient evidence that the mean is less than 3.

Hypothesis Testing Example

(continued)

- An alternate way of constructing rejection region:



$$\bar{x}_\alpha = \mu - z_\alpha \frac{\sigma}{\sqrt{n}} = 3 - 1.645\frac{0.8}{\sqrt{100}} = 2.8684$$

Since $\bar{x} = 2.84 < 2.8684$, we reject the null hypothesis

Not enough statistical evidence to conclude that the number of TVs is at least 3

p-Value Approach to Testing

- Convert Sample Statistic ($\bar{x}$) to Test Statistic (a z value, if σ is known)
- Determine the p-value from a table or computer
- Compare the p-value with ~
  - If p-value < ~ , reject H_0
  - If p-value ñ ~ , do not reject H_0

p-Value Approach to Testing

(continued)

- p-value: Probability of obtaining a test statistic more extreme (≤ or ñ) than the observed sample value given H_0 is true
  - Also called observed level of significance
  - Smallest value of ~ for which H_0 can be rejected

p-Value Approach to Testing

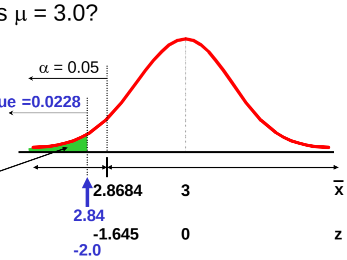
(continued)

- Adds a degree of significance to the result of the hypothesis test
- More than just a simple "reject"
  - Can now determine how strongly you "reject" or "accept"
- The further the p-value is from α, the stronger the decision

p-value Example

- Example: How likely is it to see a sample mean of 2.84 (or something further below the mean) if the true mean is μ = 3.0?

$$P(\bar{x} < 2.84 \mid \mu = 3.0)$$
$$= P\left(z < \frac{2.84 - 3.0}{0.8/\sqrt{100}}\right)$$
$$= P(z < -2.0) = 0.0228$$



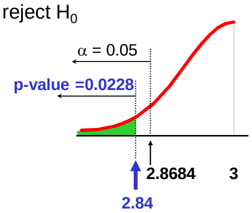
p-value Example

(continued)

- Compare the p-value with ~
  - If p-value < ~ , reject H_0
  - If p-value ñ ~ , do not reject H_0

Here: p-value = 0.0228
~ = 0.05

Since 0.0228 < 0.05, we reject the null hypothesis



Example: Upper Tail z Test for Mean (σ Known)

A phone industry manager thinks that customer monthly cell phone bill have increased, and now average over $52 per month. The company wishes to test this claim. (Assume σ = 10 is known)

Form hypothesis test:

H_0: μ ≤ 52 the average is not over $52 per month

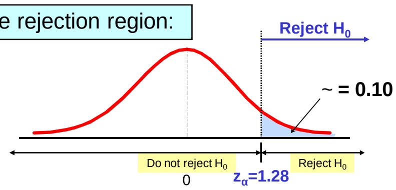
H_A: μ > 52 the average **is** greater than $52 per month (i.e., sufficient evidence exists to support the manager's claim)

Example: Find Rejection Region

(continued)

- Suppose that α = 0.10 is chosen for this test

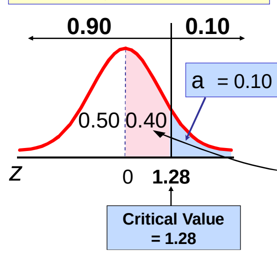
Find the rejection region:



Reject H_0 if z > 1.28

Review: Finding Critical Value - One Tail

What is z given a = 0.10?



Standard Normal Distribution Table (Portion)

Z	.07	.08	.09
1.1	.3790	.3810	.3830
1.2	.3980	.3997	.4015
1.3	.4147	.4162	.4177

Example: Test Statistic

(continued)

Obtain sample evidence and compute the test statistic

Suppose a sample is taken with the following results: n = 64, $\bar{x}$ = 53.1 (σ=10 was assumed known)

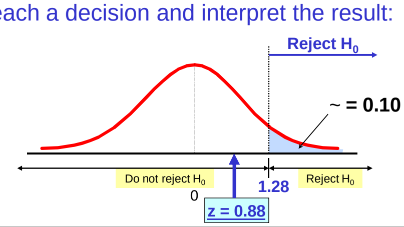
- Then the test statistic is:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = 0.88$$

Example: Decision

(continued)

Reach a decision and interpret the result:



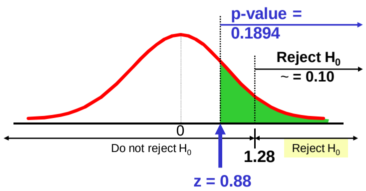
Do not reject H_0 since z = 0.88 ≤ 1.28

i.e.: there is not sufficient evidence that the mean bill is over $52

p -Value Solution

(continued)

Calculate the p-value and compare to ~



$$P(\bar{x} \geq 53.1 \mid \mu = 52.0)$$
$$= P\left(z \geq \frac{53.1 - 52.0}{10/\sqrt{64}}\right)$$
$$= P(z \geq 0.88) = 0.5 - 0.3106$$
$$= 0.1894$$

Do not reject H_0 since p-value = 0.1894 > ~ = 0.10

Critical Value Approach to Testing

- When σ is unknown, convert sample statistic ($\bar{x}$) to a t test statistic

Hypothesis Tests for «

- **± Known**
- **± Unknown**

The test statistic is:

$$t_{n-1} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

(The population must be approximately normal)

Hypothesis Tests for μ, σ Unknown

1. Specify the population value of interest
2. Formulate the appropriate null and alternative hypotheses
3. Specify the desired level of significance
4. Determine the rejection region (critical values are from the **t-distribution** with n-1 d.f.)
5. Obtain sample evidence and compute the test statistic
6. Reach a decision and interpret the result

Example: Two-Tail Test (σ Unknown)

The average cost of a hotel room in New York is said to be $168 per night. A random sample of 25 hotels resulted in $\bar{x}$ = $172.50 and s = $15.40. Test at the ~ = 0.05 level

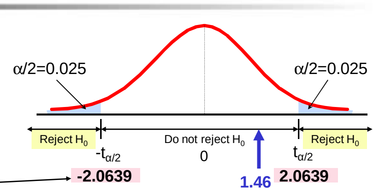
(Assume the population distribution is normal)

H_0: μ = 168
H_A: μ ÷ 168

Example Solution: Two-Tail Test

H_0: μ = 168
H_A: μ ÷ 168

- a = **0.05**
- **n = 25**
- **Critical Values: t_{24} = ± 2.0639**
- **± is unknown, so use a t statistic**



$$t_{n-1} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

Do not reject H_0: not sufficient evidence that true mean cost is different than $168

Hypothesis Tests for Proportions

- Involves categorical values
- Two possible outcomes
  - "Success" (possesses a certain characteristic)
  - "Failure" (does not possesses that characteristic)
- Fraction or proportion of population in the "success" category is denoted by π

Proportions

- The sample proportion of successes is denoted by p :

$$p = \frac{x}{n} = \frac{\text{number of successes in sample}}{\text{sample size}}$$

- When both nπ and n(1- π) are at least 5, p is approximately normally distributed with mean and standard deviation

$$\mu_p = \pi \qquad \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

Hypothesis Tests for Proportions

- The sampling distribution of p is normal, so the test statistic is a z value:

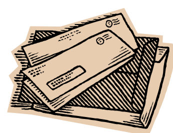
$$z = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

Hypothesis Tests for π

- **nπ ñ 5 and n(1-π) ñ 5**
- **nπ < 5 or n(1-π) < 5** → Not discussed in this chapter

Example: z Test for Proportion

A marketing company claims that it receives 8% responses from its mailing. To test this claim, a random sample of 500 were surveyed with 25 responses. Test at the ~ = 0.05 significance level.



Check:
$n\pi = (500)(0.08) = 40$
$n(1-\pi) = (500)(0.92) = 460$ ✓

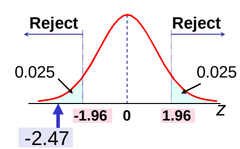
Both > 5, so assume normal

Z Test for Proportion: Solution

H0: π = 0.08
HA: π ÷ 0.08

a = 0.05
n = 500, p = 0.05

Critical Values: ± 1.96



Test Statistic:

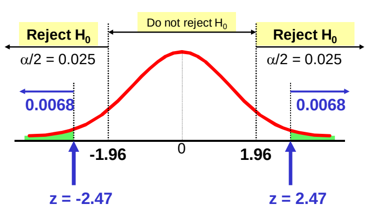
$$z = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.05-0.08}{\sqrt{\frac{0.08(1-0.08)}{500}}} = -2.47$$

Decision:
Reject H_0 at ~ = 0.05

Conclusion:
There is sufficient evidence to reject the company's claim of 8% response rate.

p -Value Solution

Calculate the p-value and compare to ~
(For a two tailed test the p-value is always two tailed)



p-value = .0136:

$$P(z \le -2.47) + P(x \ge 2.47)$$
$$= 2(0.5 - 0.4932)$$
$$= 2(0.0068) = 0.0136$$

Reject H_0 since p-value = 0.0136 < ~ = 0.05

Estimation for Two Populations

Estimating two population values

- Population means, independent samples — Group 1 vs. Group 2
- Population proportions — Proportion 1 vs. Proportion 2

Difference Between Two Means

Population means, independent samples *

- σ_1 and σ_2 known
- σ_1 and σ_2 unknown but assumed equal

Goal: Form a confidence interval for the difference between two population means, $\mu_1 - \mu_2$

The point estimate for the difference is

$$\bar{x}_1 - \bar{x}_2$$

Independent Samples

- Different data sources
  - Unrelated
  - Independent
    - Sample selected from one population has no effect on the sample selected from the other population
- Use the difference between 2 sample means

σ_1 and σ_2 known

Assumptions:

- Samples are randomly and independently drawn
- population distributions are normal or both sample sizes are ≥ 30
- Population standard deviations are known

σ_1 and σ_2 known

(continued)

When σ_1 and σ_2 are known and both populations are normal or both sample sizes are at least 30, the test statistic is a z value……and the standard error of $\bar{x}_1 - \bar{x}_2$ is

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Confidence Interval: σ_1 and σ_2 known

The confidence interval for $\mu_1 - \mu_2$ is:

$$\left(\bar{x}_1 - \bar{x}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

General Steps

1. Define the population parameter of interest and select independent samples from each population
2. Specify the confidence interval
3. Compute the point estimate
4. Determine the standard error
5. Determine the critical value
6. Develop the confidence interval estimate

σ_1 and σ_2 unknown, large samples

Population means, independent samples

- σ_1 and σ_2 known
- σ_1 and σ_2 unknown but assumed equal *

Assumptions:

- Samples are randomly and independently drawn
- Populations follow the normal distribution
- The two standard deviations are equal

σ_1 and σ_2 unknown, large samples

(continued)

Forming interval estimates:

- The population standard deviations are assumed equal, so use the two sample standard deviations and pool them to estimate σ (called pooled standard deviation)
- the test statistic is a t value with $(n_1 + n_2 - 2)$ degrees of freedom

σ_1 and σ_2 unknown, large samples

(continued)

The pooled standard deviation is:

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

The confidence interval for $\mu_1 - \mu_2$ is:

$$\left(\bar{x}_1 - \bar{x}_2\right) \pm t_{\alpha/2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Hypothesis Tests for the Difference Between Two Means

- Testing Hypotheses about $\mu_1 - \mu_2$
- Use the same situations discussed already:
  - Standard deviations known
  - Standard deviations unknown
    - Assumed equal
    - Assumed not equal (small samples)

Hypothesis Tests for Two Population Means

Two Population Means, Independent Samples

Lower tail test:	Upper tail test:	Two-tailed test:
$H_0: \mu_1$ ñ μ_2 $H_A: \mu_1 < \mu_2$	$H_0: \mu_1 \le \mu_2$ $H_A: \mu_1 > \mu_2$	$H_0: \mu_1 = \mu_2$ $H_A: \mu_1 \ne \mu_2$
i.e.,	i.e.,	i.e.,
$H_0: \mu_1 - \mu_2$ ñ 0 $H_A: \mu_1 - \mu_2 < 0$	$H_0: \mu_1 - \mu_2 \le 0$ $H_A: \mu_1 - \mu_2 > 0$	$H_0: \mu_1 - \mu_2 = 0$ $H_A: \mu_1 - \mu_2 \ne 0$

Hypothesis tests for $\mu_1 - \mu_2$

Population means, independent samples

- σ_1 and σ_2 known → Use a **z** test statistic
- σ_1 and σ_2 unknown but assumed equal → Use s_p to estimate unknown σ, use a **t** test statistic with $n_1 + n_2 - 2$ d.f.

σ_1 and σ_2 known

The test statistic for $\mu_1 - \mu_2$ is:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Steps: Hypothesis Test for Two Population Means

1. Specify parameter of interest
2. Formulate hypotheses
3. Specify the significance level (α)
4. Construct the rejection region and develop the decision rule
5. Compute the test statistics and/or the p-value
6. Reach a decision
7. Draw a conclusion

See Chapter 2

σ_1 and σ_2 unknown, large samples

The test statistic for $\mu_1 - \mu_2$ is:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Where t has $(n_1 + n_2 - 2)$ d.f.,

and

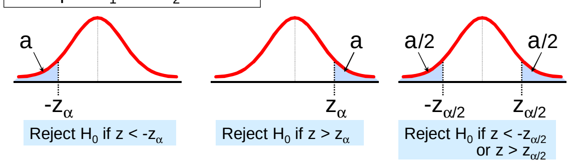
$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Hypothesis tests for $\mu_1 - \mu_2$

Two Population Means, Independent Samples

Lower tail test:	Upper tail test:	Two-tailed test:
H_0: $\mu_1 - \mu_2$ ñ 0 H_A: $\mu_1 - \mu_2 < 0$	H_0: $\mu_1 - \mu_2 \leq 0$ H_A: $\mu_1 - \mu_2 > 0$	H_0: $\mu_1 - \mu_2 = 0$ H_A: $\mu_1 - \mu_2 \neq 0$

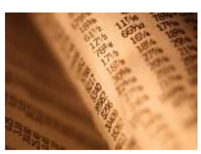
Example: σ_1 and σ_2 known:



Example
σ_1 and σ_2 unknown, assumed equal

You're a financial analyst for a brokerage firm. Is there a difference in dividend yield between stocks listed on the NYSE & NASDAQ? You collect the following data:

	NYSE	NASDAQ
Number	21	25
Sample mean	3.27	2.53
Sample std dev	1.30	1.16



Assuming equal variances, is there a difference in average yield (~ **= 0.05**)?

Calculating the Test Statistic

The test statistic is:

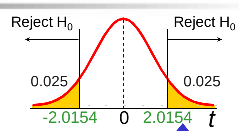
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(3.27 - 2.53) - 0}{1.2256\sqrt{\frac{1}{21} + \frac{1}{25}}} = \boxed{2.040}$$

Where:

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(21 - 1)1.30^2 + (25 - 1)1.16^2}{21 + 25 - 2}} = 1.2256$$

Solution

$H_0: \mu_1 - \mu_2 = 0$ i.e. $(\mu_1 = \mu_2)$
$H_A: \mu_1 - \mu_2 \neq 0$ i.e. $(\mu_1 \neq \mu_2)$
~ = 0.05
df = 21 + 25 - 2 = 44
Critical Values: t = ± 2.0154



Test Statistic:

$$t = \frac{3.27 - 2.53}{1.2256\sqrt{\frac{1}{21} + \frac{1}{25}}} = 2.040$$

Decision:
Reject H_0 at a = 0.05

Conclusion:
There is evidence that the means are different.

Two Population Proportions

Goal: Form a confidence interval for or test a hypothesis about the difference between two population proportions, $\pi_1 - \pi_2$

Assumptions:

$n_1\pi_1 \geq 5$, $n_1(1-\pi_1) \geq 5$

$n_2\pi_2 \geq 5$, $n_2(1-\pi_2) \geq 5$

The point estimate for the difference is $p_1 - p_2$

Confidence Interval for Two Population Proportions

The confidence interval for $\pi_1 - \pi_2$ is:

$$(p_1 - p_2) \pm z\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

Hypothesis Tests for Two Population Proportions

Population proportions

Lower tail test:	Upper tail test:	Two-tailed test:
H_0: π_1 ñ π_2 $H_A: \pi_1 < \pi_2$ i.e., H_0: $\pi_1 - \pi_2$ ñ 0 $H_A: \pi_1 - \pi_2 < 0$	$H_0: \pi_1 \leq \pi_2$ $H_A: \pi_1 > \pi_2$ i.e., $H_0: \pi_1 - \pi_2 \leq 0$ $H_A: \pi_1 - \pi_2 > 0$	$H_0: \pi_1 = \pi_2$ $H_A: \pi_1 \neq \pi_2$ i.e., $H_0: \pi_1 - \pi_2 = 0$ $H_A: \pi_1 - \pi_2 \neq 0$

Two Population Proportions

Since we begin by assuming the null hypothesis is true, we assume $\pi_1 = \pi_2$ and pool the two p estimates

The pooled estimate for the overall proportion is:

$$\bar{p} = \frac{n_1p_1 + n_2p_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$$

Where x_1 and x_2 are the numbers from samples 1 and 2 with the characteristic of interest

Two Population Proportions

(continued)

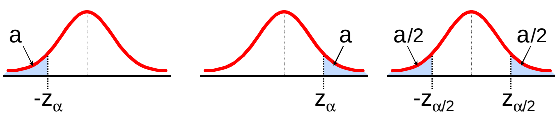
The test statistic for $\pi_1 - \pi_2$ is:

$$z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Hypothesis Tests for Two Population Proportions

Population proportions

Lower tail test:	Upper tail test:	Two-tailed test:
$H_0: \pi_1 - \pi_2$ ñ 0 $H_A: \pi_1 - \pi_2 < 0$	$H_0: \pi_1 - \pi_2 \le 0$ $H_A: \pi_1 - \pi_2 > 0$	$H_0: \pi_1 - \pi_2 = 0$ $H_A: \pi_1 - \pi_2 \ne 0$



Reject H_0 if $z < -z_\alpha$	Reject H_0 if $z > z_\alpha$	Reject H_0 if $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$

Example: Two population Proportions

Is there a significant difference between the proportion of men and the proportion of women who will vote Yes on Proposition A?

- In a random sample, 36 of 72 men and 31 of 50 women indicated they would vote Yes
- Test at the 0.05 level of significance

Example: Two population Proportions

(continued)

- The hypothesis test is:

$H_0: \pi_1 - \pi_2 = 0$ (the two proportions are equal)
$H_A: \pi_1 - \pi_2 \ne 0$ (there is a significant difference between proportions)

- The sample proportions are:

Men:	p_1 = 36/72 = 0.50
Women:	p_2 = 31/50 = 0.62

- The pooled estimate for the overall proportion is:

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{36+31}{72+50} = \frac{67}{122} = 0.549$$

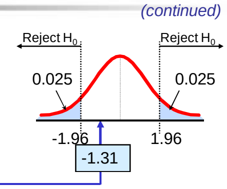
Example: Two population Proportions

(continued)

The test statistic for $\pi_1 - \pi_2$ is:

$$z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{(0.50 - 0.62) - (0)}{\sqrt{0.549\,(1 - 0.549)\left(\frac{1}{72} + \frac{1}{50}\right)}} = -1.31$$



Critical Values = ±1.96
For ~ = 0.05

Decision: Do not reject H_0

Conclusion: There is not significant evidence of a difference in the proportion who will vote yes between men and women.

Chapter Summary

- Addressed hypothesis testing methodology
- Performed z Test for the mean (σ known)
- Discussed p–value approach to hypothesis testing
- Performed one-tail and two-tail tests
- Performed t test for the mean (σ unknown)
- Performed z test for the proportion

Chapter Summary

(continued)

- Compared two independent samples
  - Formed confidence intervals for the differences between two means
  - Performed z test for the differences in two means
  - Performed *t* test for the differences in two means
- Compared two population proportions
  - Formed confidence intervals for the difference between two population proportions
  - Performed z test for two population proportions