QM353
Business Statistics

Chapter 1
Discrete and Continuous Probability Distributions

Chapter Outline

- Review of probability concepts, functions & distributions(Binomial and the Poisson pp213-34)
- The Hyper-geometric probability distribution of two possible outcomes per trial. The mean and the variance of an event of interest. (pp235-38)
- The Hyper-geometric distribution of more than two possible outcomes per trial (pp238-42)
- Continuous Uniform probability distribution: (pp270-72)
- Exponential probability distribution (pp272-77)

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Important Terms

- Probability – the chance that an uncertain event will occur (always between 0 and 1)
- Experiment – a process that produces outcomes for uncertain events
- Sample Space (or event) – the collection of all possible experimental outcomes

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Probability Concepts

- **Independent and Dependent Events**

Independent: Occurrence of one does not influence the probability of occurrence of the other

Dependent: Occurrence of one affects the probability of the other

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Rules of Probability

Individual Values

$0 \le P(E_i) \le 1$
For any event E_i

Rule 1

The value of a probability is between 0 and 1
0 = no chance of occurring
1 = 100% change of occurring

Sum of All Values

$$\sum_{i=1}^{k} P(e_i) = 1$$

Rule 2

where:
k = Number of individual outcomes in the sample space

e_i = i^{th} individual outcome

The sum of the probabilities of all the outcomes in a sample space must = 1 or 100%

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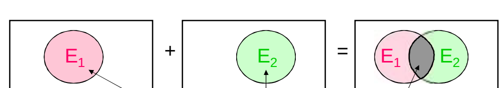
Rules of Probability
(continued)

- Addition Rule (when events are NOT mutually exclusive):

$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2)$



$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2)$

NOTE: "or" indicates addition of events

Don't count common elements twice!

joint probability

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Conditional Probability

- Conditional probability for any two events E_1 , E_2 (at the same time or in succession) :

$$P(E_1 | E_2) = \frac{P(E_1 \text{ and } E_2)}{P(E_2)}$$

Rule 6

"given that"

where $P(E_2) > 0$

e.g., Purchasing intent given that the price has been lowered

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Introduction to Probability Distributions

- **Random Variable**
  - Represents a possible numerical value from a random event
  - Can vary from trial to trial

Random Variables

- **Discrete Random Variable**
- **Continuous Random Variable**

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Discrete Probability Distribution

- A list of all possible [x_i , $P(x_i)$] pairs

  x_i = Value of Random Variable (Outcome)

  $P(x_i)$ = Probability Associated with Value

- x_i's are mutually exclusive (no overlap)
- x_i's are collectively exhaustive (nothing left out)
- $0 \le P(x_i) \le 1$ for each x_i
  - The probability of x_i is between 0 and 1
- $\Sigma P(x_i) = 1$
  - The sum of all probabilities in the sample space = 1

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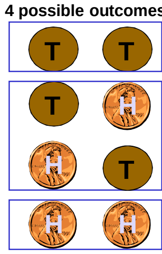
Discrete Probability Distribution

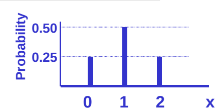
Experiment: Toss 2 Coins. Let x = # heads.

4 possible outcomes



Probability Distribution

x Value	Probability
0	1/4 = 0.25
1	2/4 = 0.50
2	1/4 = 0.25



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Discrete Random Variable Mean

- Expected Value (or mean) of a discrete distribution

  (Weighted Average)

$$E(x) = \Sigma x P(x)$$

- Example: Toss 2 coins, x = # of heads, compute expected value of x:

x	P(x)
0	0.25
1	0.50
2	0.25

E(x) = (0 x 0.25) + (1 x 0.50) + (2 x 0.25) = 1.0

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Discrete Random Variable Standard Deviation

- Standard Deviation of a discrete distribution

$$\sigma_x = \sqrt{\sum \{x - E(x)\}^2 P(x)}$$

where:

E(x) = Expected value of the random variable

x = Values of the random variable

P(x) = Probability of the random variable having the value of x

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Discrete Random Variable Standard Deviation

(continued)

- Example: Toss 2 coins, x = # heads, compute standard deviation (recall E(x) = 1)

$$\sigma_x = \sqrt{\sum \{x - E(x)\}^2 P(x)}$$

$$\sigma_x = \sqrt{(0-1)^2(0.25) + (1-1)^2(0.50) + (2-1)^2(0.25)} = \sqrt{0.50} = .707$$

Possible number of heads = 0, 1, or 2

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Probability Distributions

Discrete Probability Distributions

- Binomial
- Poisson
- Hypergeometric

Random Variable Values Are Determined by Counting

Continuous Probability Distributions

- Normal
- Uniform
- Exponential

Random Variable Values Are Determined by Measuring

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The Binomial Distribution

- Characteristics of the Binomial Distribution:
  - A trial has only two possible outcomes – “success” or “failure”
  - There is a fixed number, n (finite), of identical trials
  - The trials of the experiment are **independent** of each other
  - The probability of a success, p, remains constant from trial to trial
  - If p represents the probability of a success, then (1-p) = q is the probability of a failure

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Binomial Distribution Example

- Household Security (p. 213)

Household Security produces and installs 300 custom-made home security units every week. The units are priced to include one-day installation service by two technicians. A unit with either a design or production problem must be modified on site and will require more than one day to install.

Household Security has completed an extensive study of its design and manufacturing systems. The information shows that if the company is operating at standard quality, 10% of the security units will have problems and will require more than one day to install.

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Binomial Distribution Example

(continued)

1. There are only two possible outcomes when a unit is installed: It is good or it is defective. Finding a defective unit in this application will be considered a success.
2. Each unit is designed and made in the same way.
3. The outcome of a security unit (good or defective) is independent of whether the preceding unit was good or defective.
4. The probability of a defective unit, *p = 0.10, remains* from unit to unit

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Binomial Distribution Example

(continued)

Results	No. of Defectives	No. of Ways
G,G,G,G	0	1
G,G,G,D	1	
G,G,D,G		4
G,D,G,G		
D,G,G,G		
G,G,D,D	2	
G,D,G,D		
D,G,G,D		6
G,D,D,G		
D,G,D,G		
D,D,G,G		
D,D,D,G	3	
D,D,G,D		4
D,G,D,D		
G,D,D,D		
D,D,D,D	4	1

P(G and G and G and D)
= *P(G)P(G)P(G)P(D)*
=*(0.90)*(0.90)(0.90)(0.10)
= $(0.90)^3(0.10)$
Likewise:
P(G and G and D and G)
=*P(G and D and G and G)*
=*P(D and G and G and G)*
= $(0.90)^3(0.10)$

$$P(1 \text{ defective}) = P(G \text{ and } G \text{ and } G \text{ and } D) + P(G \text{ and } G \text{ and } D \text{ and } G) + P(G \text{ and } D \text{ and } G \text{ and } G) + P(D \text{ and } G \text{ and } G \text{ and } G)$$

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Counting Rule for Combinations

- A combination is an outcome of an experiment where x objects are selected from a group of n objects

$$C_x^n = \frac{n!}{x!(n-x)!}$$

where:

C_x^n = number of combinations of x objects selected from n objects
n! =n(n - 1)(n - 2) . . . (2)(1)
x! = x(x - 1)(x - 2) . . . (2)(1)

NOTE: 0! = 1 (by definition)

Order does not matter
i.e. ABC = CBA only one outcome

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Binomial Distribution Formula

$$P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

P(x) = probability of **x** successes in **n** trials, with probability of success **p** on each trial

x = number of *successes* in sample, (x = 0, 1, 2, ..., n)
p = probability of "success" per trial
q = probability of "failure" = (1 – p)
n = number of trials (sample size)

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Appendix B

N = 4

K \ P=	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.6561	0.4096	0.2401	0.1296	0.0625	0.0256	0.0081	0.0016	0.0001
1	0.2916	0.4096	0.4116	0.3456	0.2500	0.1536	0.0756	0.0256	0.0036
2	0.0486	0.1536	0.2646	0.3456	0.3750	0.3456	0.2646	0.1536	0.0486
3	0.0036	0.0256	0.0756	0.1536	0.2500	0.3456	0.4116	0.4096	0.2916
4	0.0001	0.0016	0.0081	0.0256	0.0625	0.1296	0.2401	0.4096	0.6561

N = 5

K \ P=	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.59049	0.32768	0.16807	0.07776	0.03125	0.01024	0.00243	0.00032	0.00001
1	0.32805	0.40960	0.36015	0.25920	0.15625	0.07680	0.02835	0.00640	0.00045
2	0.07290	0.20480	0.30870	0.34560	0.31250	0.23040	0.13230	0.05120	0.00810
3	0.00810	0.05120	0.13230	0.23040	0.31250	0.34560	0.30870	0.20480	0.07290
4	0.00045	0.00640	0.02835	0.07680	0.15625	0.25920	0.36015	0.40960	0.32805
5	0.00001	0.00032	0.00243	0.01024	0.03125	0.07776	0.16807	0.32768	0.59049

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Binomial Distribution Summary Measures

- Mean

$$\mu = E(x) = np$$

- Variance and Standard Deviation

$$\sigma^2 = npq$$

$$\sigma = \sqrt{npq}$$

Where n = sample size
p = probability of success
q = (1 – p) = probability of failure

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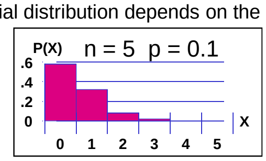
Binomial Distribution

- The shape of the binomial distribution depends on the values of p and n

- Here, n = 5 and p = 0.1

$\mu = np = (5)(0.1) = 0.5$

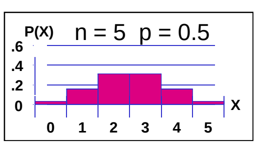
$\sigma = \sqrt{npq} = \sqrt{(5)(0.1)(1-.1)}$
$= 0.6708$



- Here, n = 5 and p = 0.5

$\mu = np = (5)(0.5) = 2.5$

$\sigma = \sqrt{npq} = \sqrt{(5)(0.5)(1-0.5)}$
$= 1.118$



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Binomial Distribution Example

- Example: 35% of all voters support Proposition A. If a random sample of 10 voters is polled, what is the probability that exactly three of them support the proposition?

i.e., find P(x = 3) if n = 10 and p = 0.35 :

$$P(x=3) = \frac{n!}{x!(n-x)!} p^x q^{n-x} = \frac{10!}{3!7!}(0.35)^3(0.65)^7 = 0.2522$$

There is a 25.22% chance that exactly 3 out of the 10 voters will support Proposition A

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The Poisson Distribution

- Characteristics of the Poisson Distribution:
  - The average number of outcomes of interest per time or space interval is a
  - The number of outcomes of interest are random, and the occurrence of one outcome does not influence the chances of another outcome of interest
  - The probability that an outcome of interest occurs in a given segment is the same for all segments
  - We imagine dividing time or space into tiny subsegments. Then the chance of *more than one success in a subsegment* is negligible and the chance of exactly one success in a tiny subsegment of length *t is λ t.*

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Poisson Distribution Formula

$$P(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

where:
- t = size of the segment of interest
- x = number of successes in segment of interest
- λ = expected number of successes in a segment of unit size
- e = base of the natural logarithm system (2.71828...)

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Using Poisson Tables

	a t								
X	0.10	0.20	0.30	0.40	**0.50**	0.60	0.70	0.80	0.90
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659
2	0.0045	0.0164	0.0333	0.0536	**0.0758**	0.0988	0.1217	0.1438	0.1647
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494
4	0.0000	0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111
5	0.0000	0.0000	0.0000	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Example: Find P(x = 2) if λ = 3 and t = 1/6 (10 Minutes)

$$P(x=2) = \frac{({}^{a}t)^x e^{-{}^{a}t}}{x!} = \frac{(0.50)^2 e^{-0.50}}{2!} = 0.0758$$

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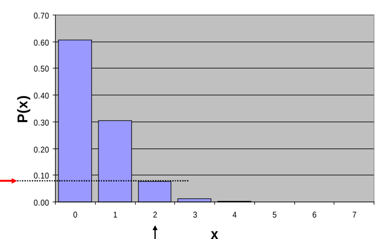
Graph of Poisson Probabilities

Graphically:

a = 3 and t = 1/6

X	a t = 0.50
0	0.6065
1	0.3033
2	0.0758
3	0.0126
4	0.0016
5	0.0002
6	0.0000
7	0.0000



P(x = 2) = 0.0758

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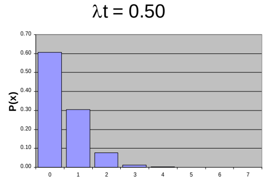
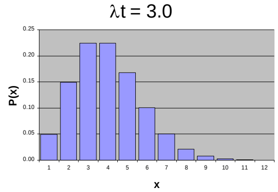
Poisson Distribution Shape

- The shape of the Poisson Distribution depends on the parameters λ and t:

λt = 0.50



λt = 3.0



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Poisson Distribution Summary Measures

- Mean

$$\mu = \lambda t$$

- Variance and Standard Deviation

$$\sigma^2 = \lambda t$$

$$\sigma = \sqrt{\lambda t}$$

where λ = number of successes in a segment of unit size
t = the size of the segment of interest

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Using the Poisson Distribution

- **Define the segment units.** The segment units are usually blocks of time, areas of space, or volume.
- **Determine the mean of the random variable.** The mean is the parameter that defines the Poisson distribution and is referred to as λ. *It is the average* number of successes in a segment of unit size.
- **Determine *t, the number of the segments*** to be considered, and then calculate λt.
- **Define the event of interest** and use **the Poisson formula** or the **Poisson table** to find the probability.

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Exercise: Poisson Distribution

Exercise 5-50 (p. 240)

Arrivals to a bank automated teller machine (ATM) are distributed according to a Poisson distribution with a mean equal to three per 15 minutes.

a. Determine the probability that in a given 15-minute segment no customers will arrive at the ATM.

b. What is the probability that fewer than four customers will arrive in a 30-minute segment?

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Exercise: Poisson Distribution

(continued)

Step 1: **Define the segment unit**. Because the mean was stated to be 3 arrivals per 15 minutes, the segment unit is 15 minutes or .25 hours.

Step 2: **Determine the mean of the random variable**. The mean is $\lambda=3$

Step 3: **Determine the segment size, t and then calculate λt**. The issue in the problem asks for the probability of no customers arriving in 15 minutes which is one segment so t = 1. Thus, $\lambda t=3$

Step 4: **Define the event of interest and use the Poisson table to find the desired probability**. The event of interest is: P(x = 0). To use the Poisson table, go to the column headed . Then find the value of x from the left hand column. The desired probability is: P(x = 0) = 0.0498

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Exercise: Poisson Distribution

(continued)

Step 1: **Define the segment unit**. Because the mean was stated to be 3 arrivals per 15 minutes, the segment unit is 15 minutes or .25 hours.

Step 2: **Determine the mean of the random variable**. The mean is $\lambda=3$

Step 3: **Determine the segment size, t and then calculate λt**. The issue in the problem asks for the probability of fewer than 3 customers arriving in 30 minutes which is two segments so t = 2. Thus, $\lambda t=6$

Step 4: **Define the event of interest and use the Poisson table to find the desired probability**. We are asked to calculate the probability that fewer than 4 customers will arrive. Thus, the event of interest is: P(x <= 3). To use the Poisson table, go to the column headed . Then find the values of x from the left hand column. The desired probability is: P(x <= 3) = 0.1512

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The Hypergeometric Distribution

- "n" trials in a sample taken from a finite population of size N
- Sample taken without replacement
- Trials are dependent
- The probability changes from trial to trial
- Concerned with finding the probability of "x" successes in the sample where there are "X" successes in the population

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Hypergeometric Distribution Example

- Example: 3 Light bulbs were selected from 10. Of the 10 there were 4 defective. What is the probability that none of the 3 selected are defective?

G G G

$$\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = 0.166$$

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Hypergeometric Distribution Formula

(Two possible outcomes per trial: success or failure)

$$P(x) = \frac{C_{n-x}^{N-X} \cdot C_x^X}{C_n^N}$$

Where

N = population size
X = number of successes in the population
n = sample size
x = number of successes in the sample
n – x = number of failures in the sample

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Hypergeometric Distribution Example

(continued)

- Example: 3 Light bulbs were selected from 10. Of the 10 there were 4 defective. What is the probability that none of the 3 selected are defective?

N = 10	n = 3
X = 4	x = 0

$$P(x=0) = \frac{C_{n-x}^{N-X}\ C_x^X}{C_n^N} = \frac{C_3^6\ C_0^4}{C_3^{10}} = \frac{(20)(1)}{120} = 0.166$$

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Exercise: Hypergeometric Distribution

Exercise 5-52 (p. 240)

A population of 10 items contains 3 that are red and 7 that are green. What is the probability that in a random sample of 3 items selected without replacement, 2 red and 1 green items are selected?

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Exercise: Hypergeometric Distribution

(continued)

- To determine this probability we recognize that because the sampling is without replacement and the sample size is large relative to the size of the population, the hypergeometric distribution applies. The following steps can be used:

Step 1: **Define the population size and the combined sample size.**

The population size is N = 10 and the combined sample size is n = 3

Step 2: **Define the event of interest.**

We are interested in the event described by getting

$$P(x = 2\ red\ and\ n - x = 1\ green) = ?$$

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Exercise: Hypergeometric Distribution

(continued)

Step 3: **Determine the number of each category in the population.**

The population contains X = 3 red and N-X = 7 green.

Step 4: **Compute the desired probability using the hypergeometric distribution.**

$$P(x=2) = \frac{C_{n-x}^{N-X} * C_x^X}{C_n^N} = \frac{C_{3-2}^{10-3} * C_2^3}{C_3^{10}} = \frac{C_1^7 * C_2^3}{C_3^{10}} = \frac{(7)(3)}{120} = \frac{21}{120} = 0.175$$

Thus, the probability of selecting 2 red and 1 green from a population with 3 red and 7 green is 0.175.

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Hypergeometric Distribution with more than two possible Outcomes

$$P(x_1, x_2, ..., x_k) = \frac{C_{x_1}^{X_1} \times C_{x_2}^{X_2} \times ... \times C_{x_k}^{X_k}}{C_n^N}$$

Where

$$\sum_{i=1}^{k} X_i = N \text{ and } \sum_{i=1}^{k} x_i = n$$

N = population size
X_i = Number of items in the population with outcome i
n = sample size
x_i = Number of items in the sample with outcome i

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Exercise: Hypergeometric Distribution with more than two possible Outcomes

Exercise 5-53 (p. 240)

Consider a situation in which a used-car lot contains five Fords, four General Motors (GM) cars, and five Toyotas. If five cars are selected at random to be placed on a special sale, what is the probability that three are Fords and two are GMs?

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Exercise: Hypergeometric Distribution with more than two possible Outcomes

$$P(x_1 = 3, x_2 = 2, x_3 = 0) = \frac{C_{x_1}^{X_1} * C_{x_2}^{X_2} * C_{x_3}^{X_3}}{C_n^N}$$

$$= \frac{C_3^5 * C_2^4 * C_0^5}{C_5^{14}}$$

$$= \frac{(10)(6)(1)}{2,002} = \frac{60}{2,002}$$

$$= 0.03$$

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Continuous Probability Distributions

- A continuous random variable is a variable that can assume any value on a defined continuum (can assume an uncountable number of values)
  - thickness of an item
  - time required to complete a task
- These can potentially take on any value, depending only on the ability to **measure** accurately.

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Types of Continuous Distributions

- Three types
  - Normal
  - Uniform
  - Exponential

A ⟶ B

Involves determining the probability for a RANGE of values rather than 1 particular incident or outcome

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The Normal Distribution

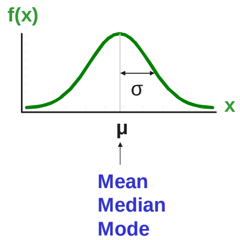
- **Bell Shaped**
- **Symmetrical**
- **Mean=Median=Mode**

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

Location is determined by the mean, μ

Spread is determined by the standard deviation, σ

The random variable has an infinite theoretical range:
+ ã to J ã



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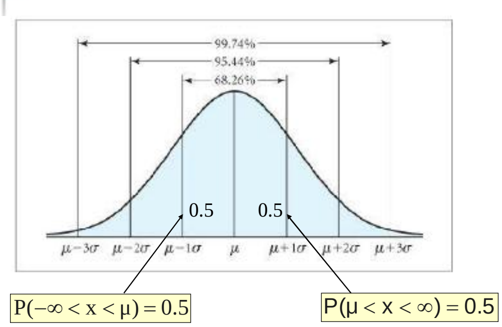
Many Normal Distributions



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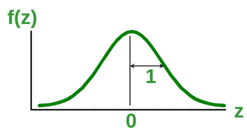
The Standard Normal Distribution

- Also known as the "z" distribution
- Mean is defined to be 0
- Standard Deviation is 1



Values above the mean have **positive** z-values
Values below the mean have **negative** z-values

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Translation to the Standard Normal Distribution

- Translate from x to the standard normal (the "z" distribution) by subtracting the mean of x and dividing by its standard deviation:

$$z = \frac{x - \mu}{\sigma}$$

z is the number of standard deviations units that x is away from the population mean

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Example

- If x is distributed normally with mean of 100 and standard deviation of 50, the z value for x = 250 is

$$z = \frac{x - \mu}{\sigma} = \frac{250 - 100}{50} = 3.0$$

- This says that x = 250 is three standard deviations (3 increments of 50 units) above the mean of 100.

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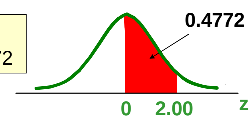
The Standard Normal Table

- The Standard Normal table in the textbook (Appendix D)
- Gives the probability from the mean (zero) up to a desired value for z

Example:
P(0 < z < 2.00) = 0.4772



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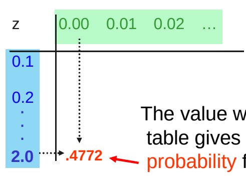
The Standard Normal Table

(continued)

The column gives the value of z to the second decimal point

The row shows the value of z to the first decimal point



The value within the table gives the probability from z = 0 up to the desired z value

P(0 < z < 2.00) = 0.4772

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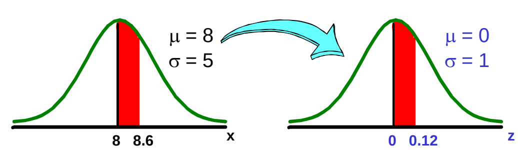
z Table Example

- Suppose x is normal with mean 8.0 and standard deviation 5.0. Find P(8 < x < 8.6)



P(8 < x < 8.6) P(0 < z < 0.12)

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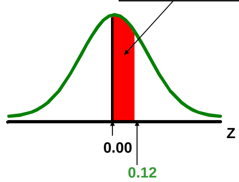
Solution: Finding P(0 < z < 0.12)

Standard Normal Probability Table (Portion)

z	.00	.01	.02
0.0	.0000	.0040	.0080
0.1	.0398	.0438	.0478
0.2	.0793	.0832	.0871
0.3	.1179	.1217	.1255

P(8 < x < 8.6)
= P(0 < z < 0.12)
0.0478



6-55

The Uniform Distribution

The uniform distribution is a probability distribution that has equal probabilities for all possible outcomes of the random variable

- Referred to as the distribution of "little information"
- Probability is the same for ANY interval of the same width
- Useful when you have limited information about how the data "behaves"

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The Uniform Distribution

(continued)

The Continuous Uniform Distribution:

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

where
(x) = value of the density function at any x value
a = lower limit of the interval of interest
b = upper limit of the interval of interest

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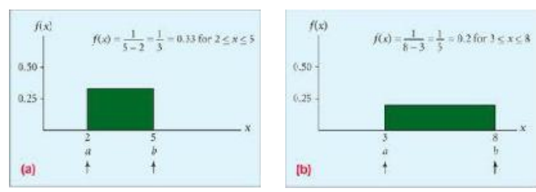
The Uniform Distribution

(continued)



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The Mean and Standard Deviation for the Uniform Distribution

The mean (expected value) is:

$$E(x) = \mu = \frac{a+b}{2}$$

The standard deviation is

$$\sigma = \sqrt{\frac{(b-a)^2}{12}}$$

where
a = lower limit of the interval from a to b
b = upper limit of the interval from a to b

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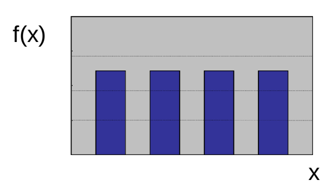
Steps for Using the Uniform Distribution

1. Define the density function
2. Define the event of interest
3. Calculate the required probability



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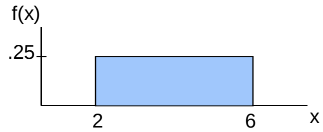
Uniform Distribution

Example: Uniform Probability Distribution
Over the range $2 \le x \le 6$:

$$f(x) = \frac{1}{6 - 2} = .25 \quad \text{for } 2 \le x \le 6$$



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Uniform Distribution

Example: Uniform Probability Distribution
Over the range $2 \le x \le 6$:

$$E(x) = \mu = \frac{2+6}{2} = 4$$

$$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(6-2)^2}{12}} = 1.1547$$

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Exercise: Uniform Distribution

- **Exercise 6-38** (p. 275)
  When only the value-added time is considered, the time it takes to build a laser printer is thought to be uniformly distributed between 8 and 15 hours.

a. What are the chances that it will take more than 10 valueadded hours to build a printer?

b. How likely is it that a printer will require less than 9 value-added hours?

c. Suppose a single customer orders two printers. Determine the probability that the first and second printer each will require less than 9 value-added hours to complete.

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Exercise: Uniform Distribution

(continued)

a) P(x>10) = (15-10)/(15-8) = 5/7 = 0.7143

b) P(x<9) = (9-8)/(15-8) = 1/7 = 0.1429

c) (0.1429)(0.1429) = 0.0204

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The Exponential Distribution

- Used to measure the time that elapses between two occurrences of an event (the time between arrivals)
  - Examples:
    - Time between trucks arriving at a dock
    - Time between transactions at an ATM Machine
    - Time between phone calls to the main operator
  - Recall λ = mean for Poisson

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The Exponential Distribution

- The probability that an arrival time is equal to or less than some specified time a is

$$P(0 \le x \le a) = 1 - e^{-\lambda a}$$

where $1/\lambda$ is the mean time between events and e = 2.7183

NOTE: If the number of occurrences per time period is Poisson with mean λ, then the time between occurrences is exponential with mean time $1/\lambda$ and the standard deviation also is $1/\lambda$

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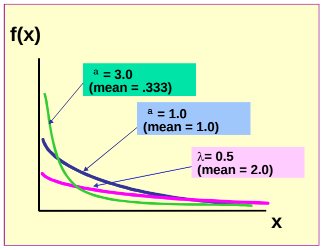
Exponential Distribution

(continued)

- Shape of the exponential distribution



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Example

Example: Customers arrive at the claims counter at the rate of 15 per hour (Poisson distributed). What is the probability that the arrival time between consecutive customers is less than five minutes?

- Time between arrivals is exponentially distributed with mean time between arrivals of 4 minutes (15 per 60 minutes, on average)
- $1/\lambda = 4.0$, so $\lambda = .25$
- $P(x < 5) = 1 - e^{-\lambda a} = 1 – e^{-(.25)(5)} =$ 0.7135

There is a 71.35% chance that the arrival time between consecutive customers is less than 5 minutes

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Exercise: Exponential Distribution

- **Exercise 6-42** (p. 276)

A delicatessen located in the heart of the business district of a large city serves a variety of customers. The delicatessen is open 24 hours a day every day of the week. In an effort to speed up take-out orders, the deli accepts orders by fax. If, on the average, 20 orders are received by fax every two hours throughout the day, find the

a. probability that a faxed order will arrive within the next 9 minutes

b. probability that the time between two faxed orders will be between 3 and 6 minutes

c. probability that 12 or more minutes will elapse between faxed orders

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Exercise: Exponential Distribution

(continued)

a. The problem is easier if times are converted to minutes. If, on the average, 20 orders are received every two hours by fax then 20/120 = 0.1667 orders are received every minute by fax. The probability that a faxed order will arrive within the next 9 minutes is equal to $1- e^{-\lambda a}$, wich is equal to $1- e^{-0.1667(9)}=0.7769$

b. $P(3 \leq x \leq 6)$ $e^{-0.1667(3)} - e^{-0.1667(6)}$ 0.6065 0.3679 0.2386.

c. $P(x > 12)$ $e^{-\lambda a}$ $e^{-0.1667(12)}$ 0.1353

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Chapter Summary

- Reviewed key discrete distributions
  - Binomial, Poisson, Hypergeometric
  - Normal, uniform, exponential
- Found probabilities using formulas and tables
- Recognized when to apply different distributions
- Applied distributions to decision problems

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