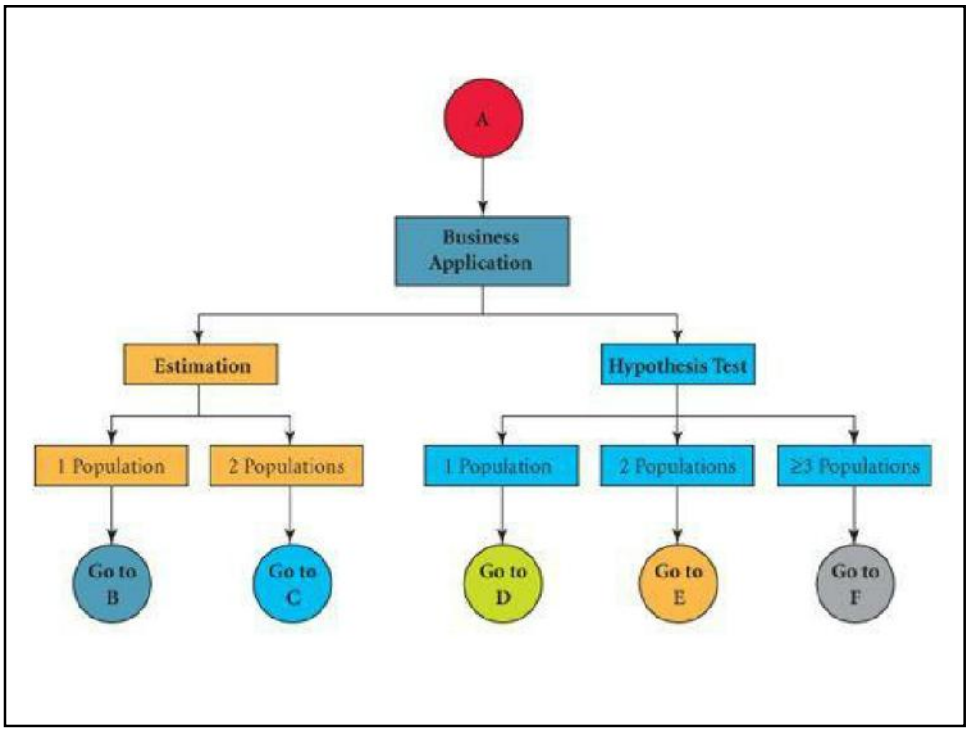
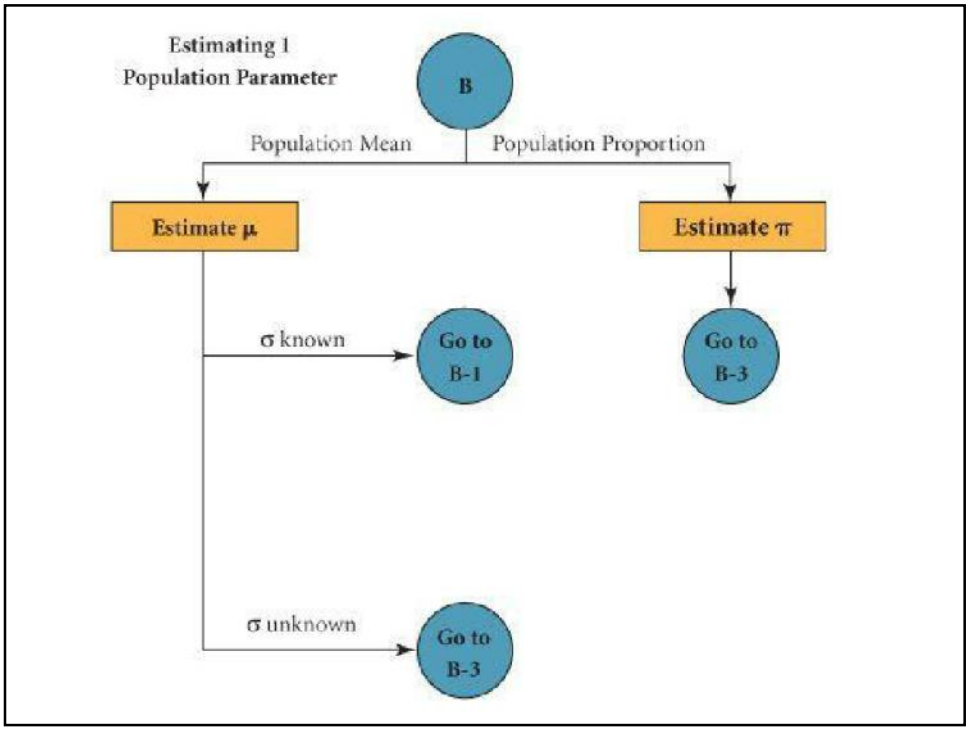
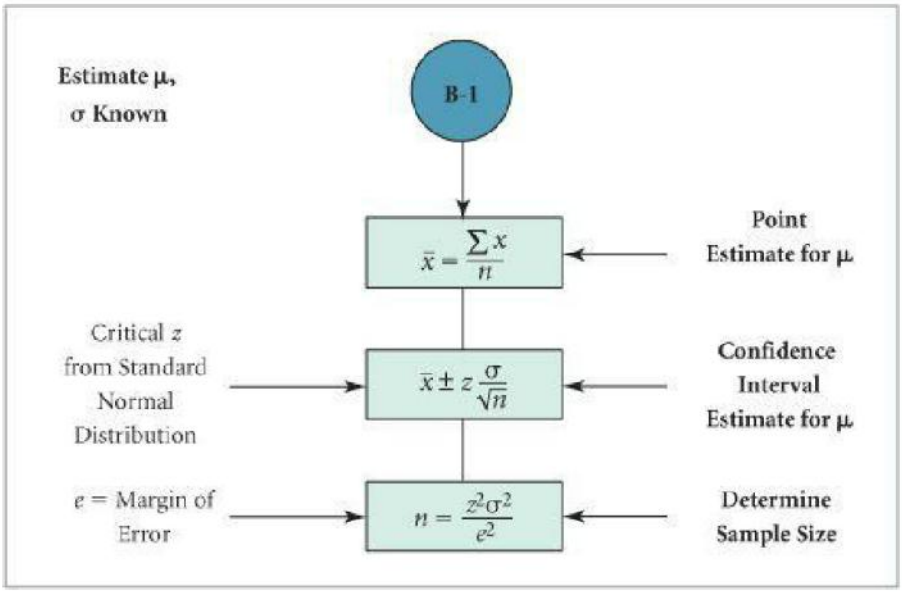
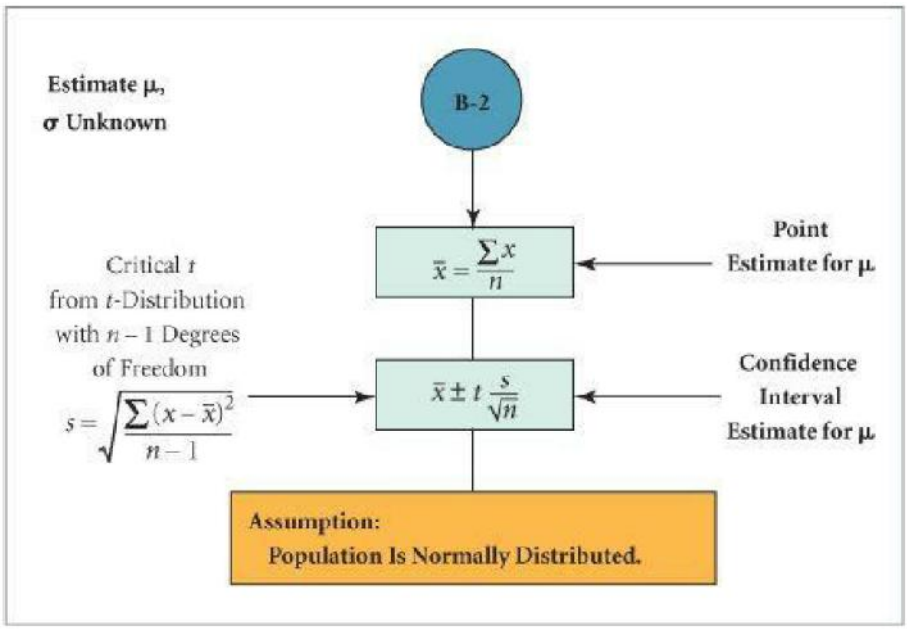
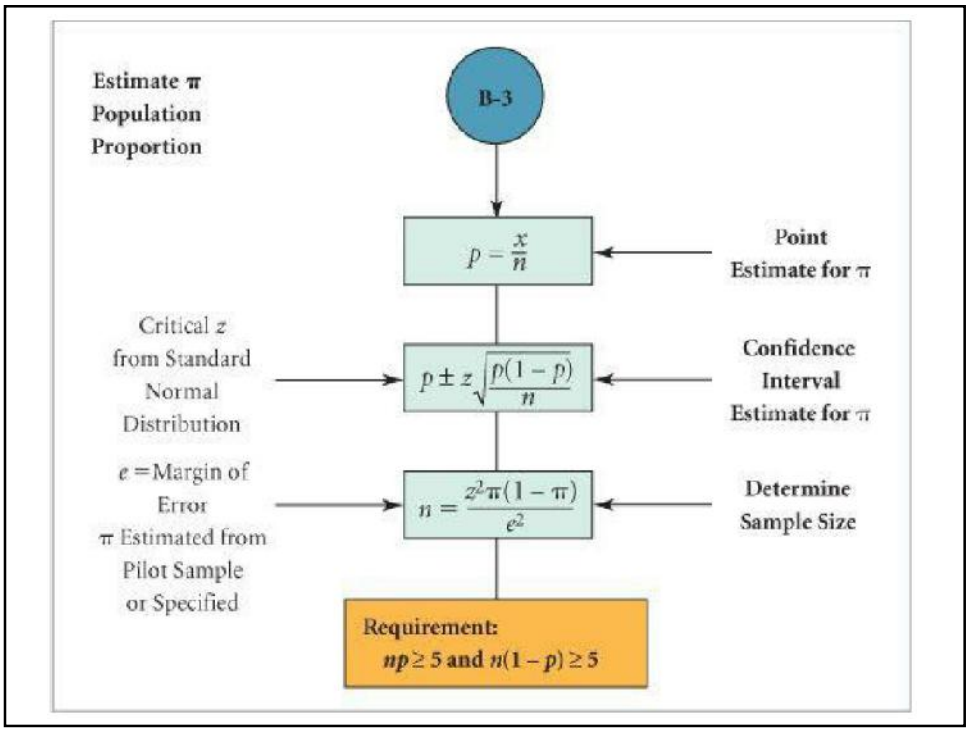
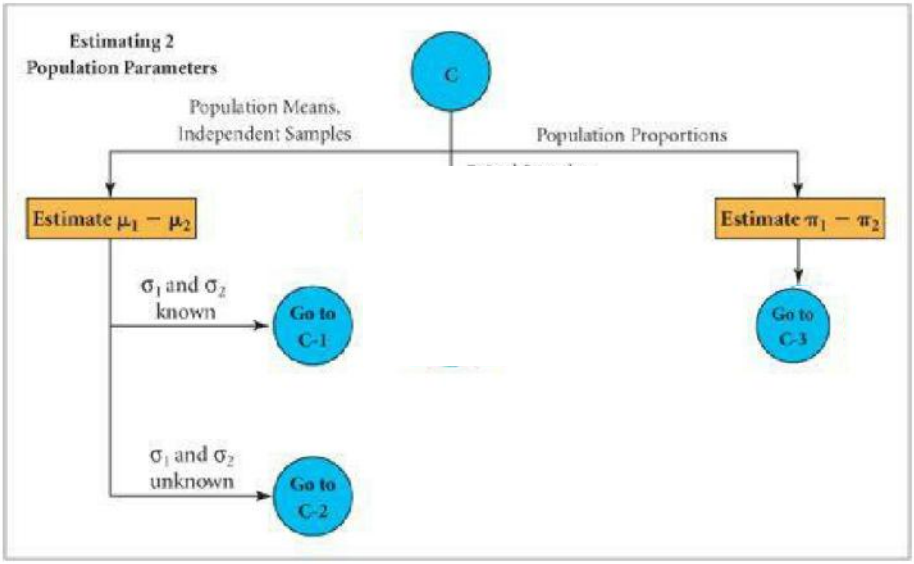

A
Business Application
Estimation
Hypothesis Test
1 Population
2 Populations
1 Population
2 Populations
≥3 Populations
Go to B
Go to C
Go to D
Go to E
Go to F

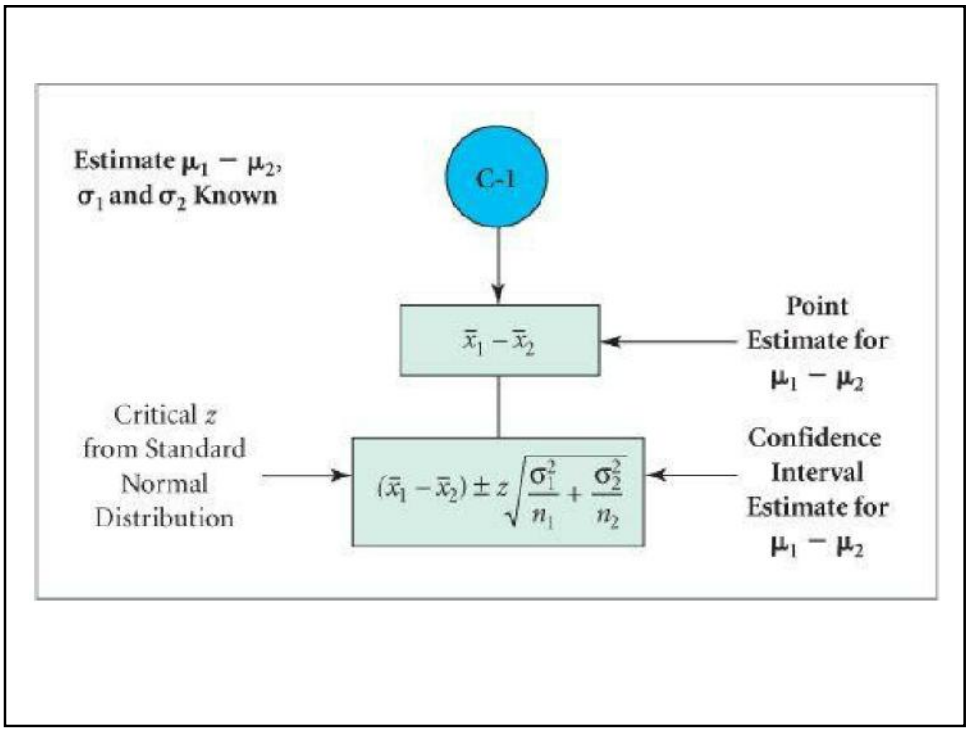

Estimating 1 Population Parameter
B
Population Mean
Population Proportion
Estimate μ
Estimate π
σ known
Go to B-1
Go to B-3
σ unknown
Go to B-3

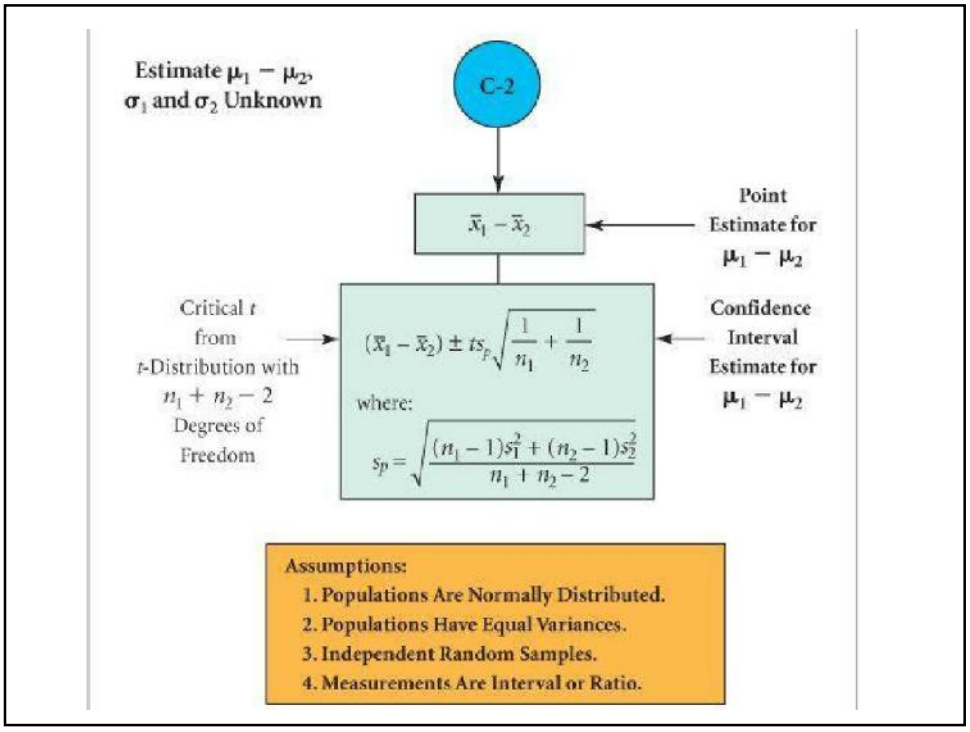

Estimate μ,
σ Known
B-1
$\bar{x} = \frac{\sum x}{n}$
Point Estimate for μ
Critical z from Standard Normal Distribution
$\bar{x} \pm z \frac{\sigma}{\sqrt{n}}$
Confidence Interval Estimate for μ
e = Margin of Error
$n = \frac{z^2\sigma^2}{e^2}$
Determine Sample Size

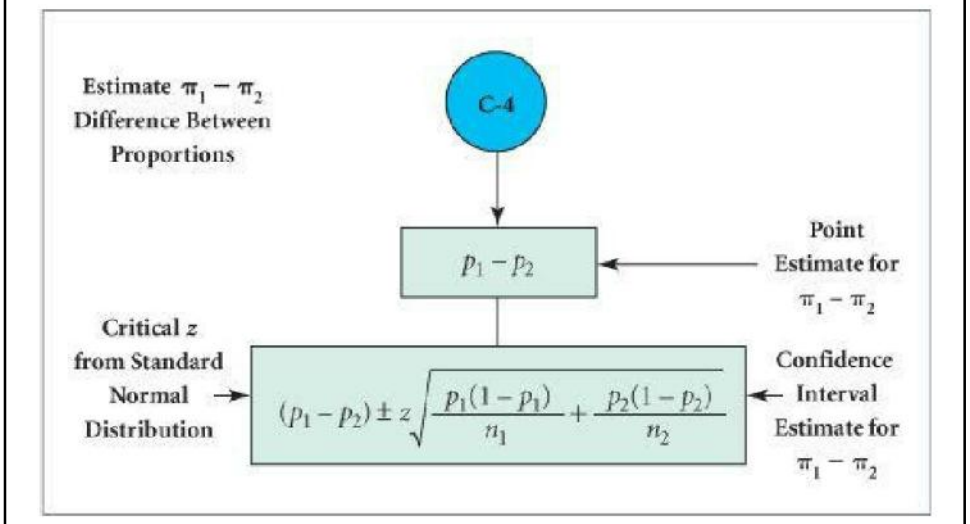

Estimate μ,
σ Unknown
B-2
$\bar{x} = \frac{\sum x}{n}$
Point Estimate for μ
Critical t from t-Distribution with n − 1 Degrees of Freedom
$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$
$\bar{x} \pm t \frac{s}{\sqrt{n}}$
Confidence Interval Estimate for μ
Assumption:
Population Is Normally Distributed.

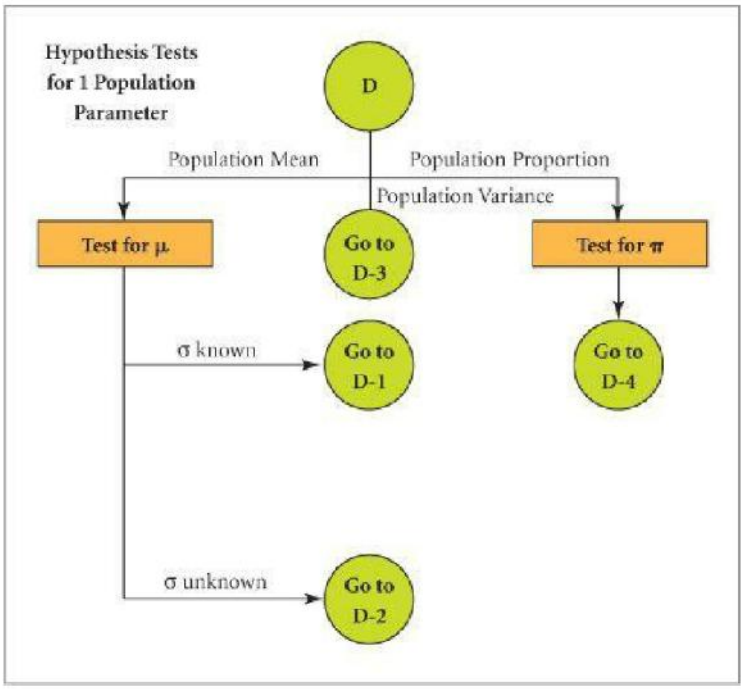

Estimate π Population Proportion
B-3
$p = \frac{x}{n}$
Point Estimate for π
Critical z from Standard Normal Distribution
$p \pm z\sqrt{\frac{p(1-p)}{n}}$
Confidence Interval Estimate for π
e = Margin of Error
π Estimated from Pilot Sample or Specified
$n = \frac{z^2\pi(1-\pi)}{e^2}$
Determine Sample Size
Requirement:
$np \geq 5$ and $n(1-p) \geq 5$

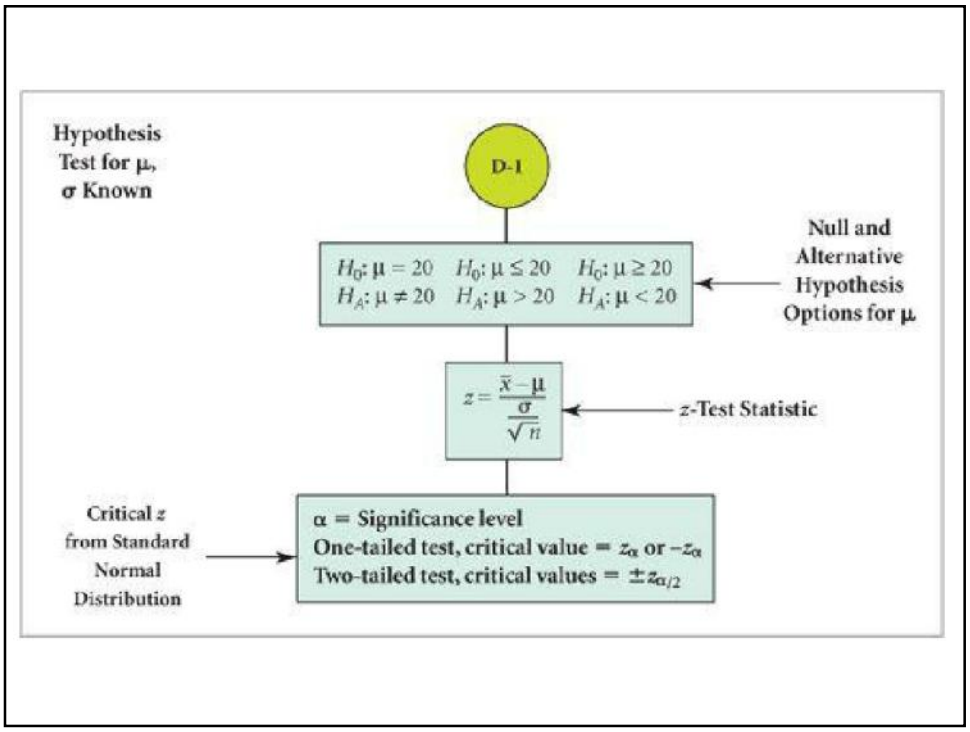

Estimating 2 Population Parameters
C
Population Means, Independent Samples
Population Proportions
Estimate $\mu_1 - \mu_2$
Estimate $\pi_1 - \pi_2$
σ_1 and σ_2 known
Go to C-1
Go to C-3
σ_1 and σ_2 unknown
Go to C-2

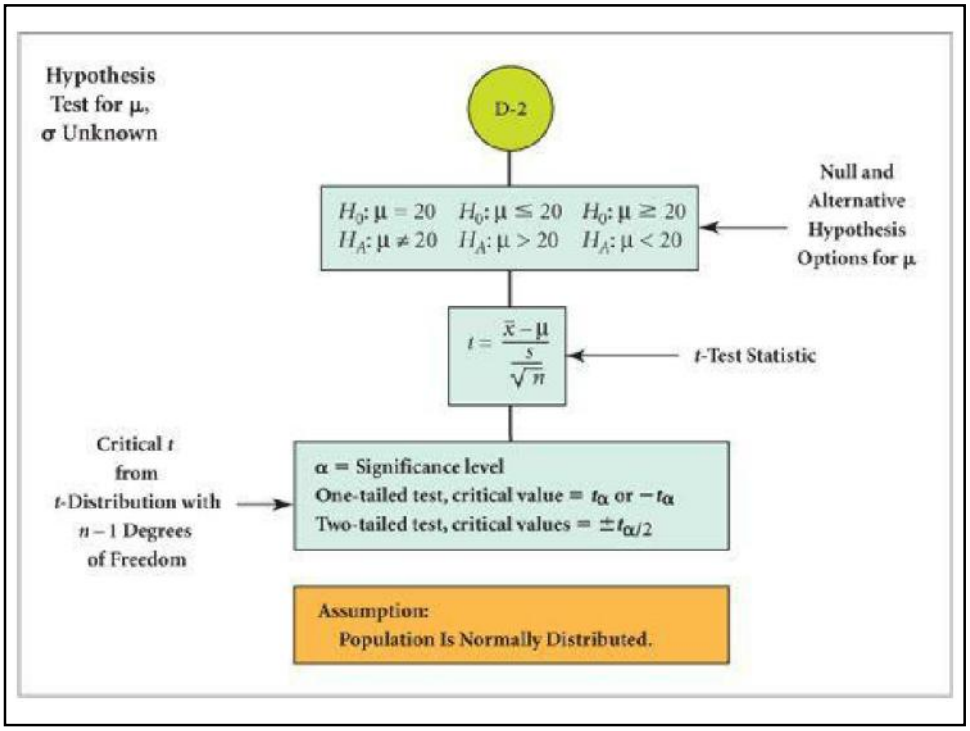

Estimate $\mu_1 - \mu_2$, σ_1 and σ_2 Known
C-1
$\bar{x}_1 - \bar{x}_2$
Point Estimate for $\mu_1 - \mu_2$
Critical z from Standard Normal Distribution
$(\bar{x}_1 - \bar{x}_2) \pm z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
Confidence Interval Estimate for $\mu_1 - \mu_2$

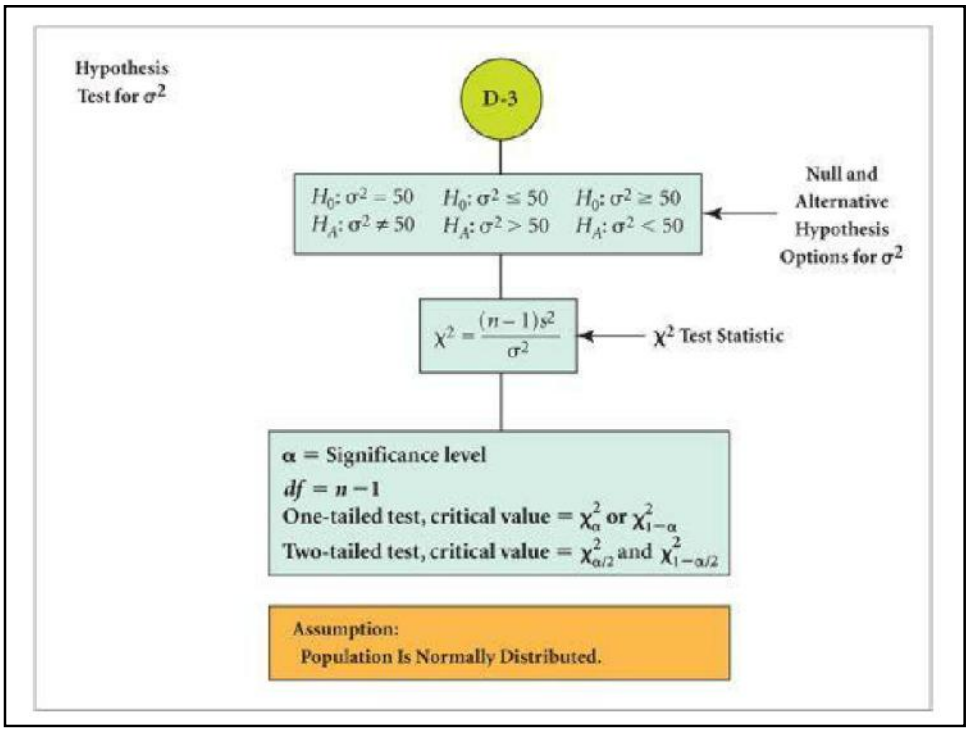

Estimate $\mu_1 - \mu_2$, σ_1 and σ_2 Unknown
C-2
$\bar{x}_1 - \bar{x}_2$
Point Estimate for $\mu_1 - \mu_2$
Critical t from t-Distribution with $n_1 + n_2 - 2$ Degrees of Freedom
$(\bar{x}_1 - \bar{x}_2) \pm t s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
where:
$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$
Confidence Interval Estimate for $\mu_1 - \mu_2$
Assumptions:
1. Populations Are Normally Distributed.
2. Populations Have Equal Variances.
3. Independent Random Samples.
4. Measurements Are Interval or Ratio.

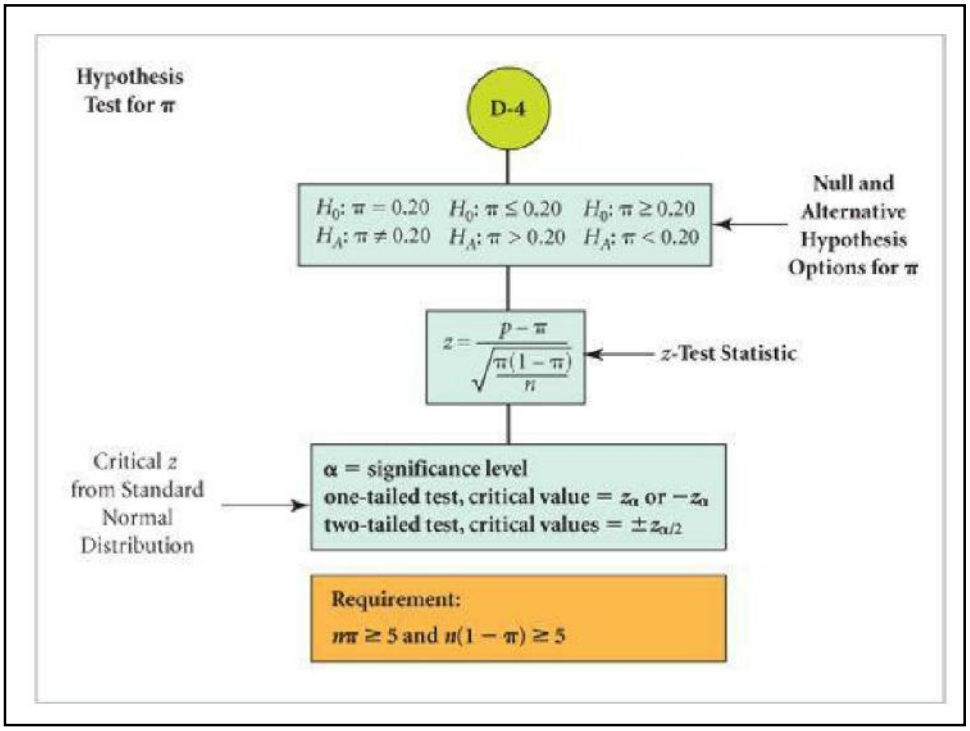

Estimate $\pi_1 - \pi_2$ Difference Between Proportions
C-4
$p_1 - p_2$
Point Estimate for $\pi_1 - \pi_2$
Critical z from Standard Normal Distribution
$(p_1 - p_2) \pm z\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$
Confidence Interval Estimate for $\pi_1 - \pi_2$

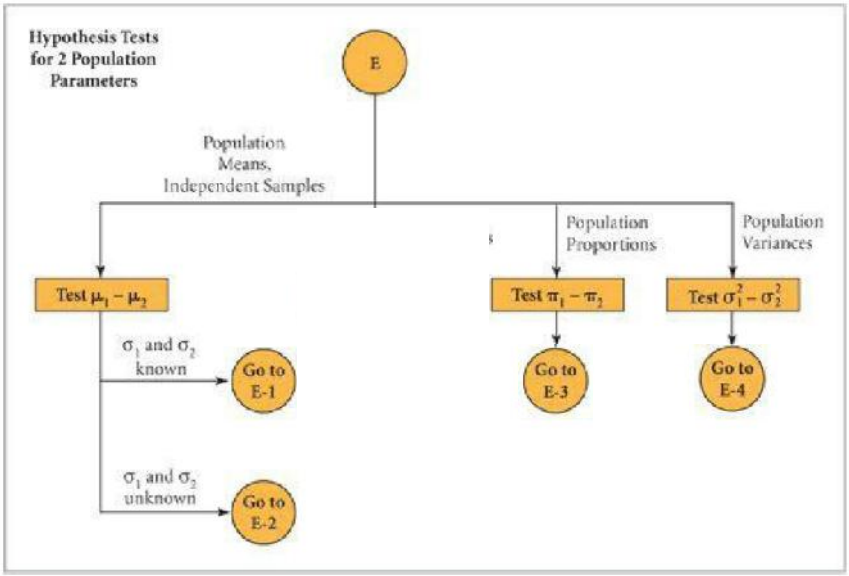

Hypothesis Tests for 1 Population Parameter
D
Population Mean
Population Proportion
Population Variance
Test for μ
Go to D-3
Test for π
σ known
Go to D-1
Go to D-4
σ unknown
Go to D-2

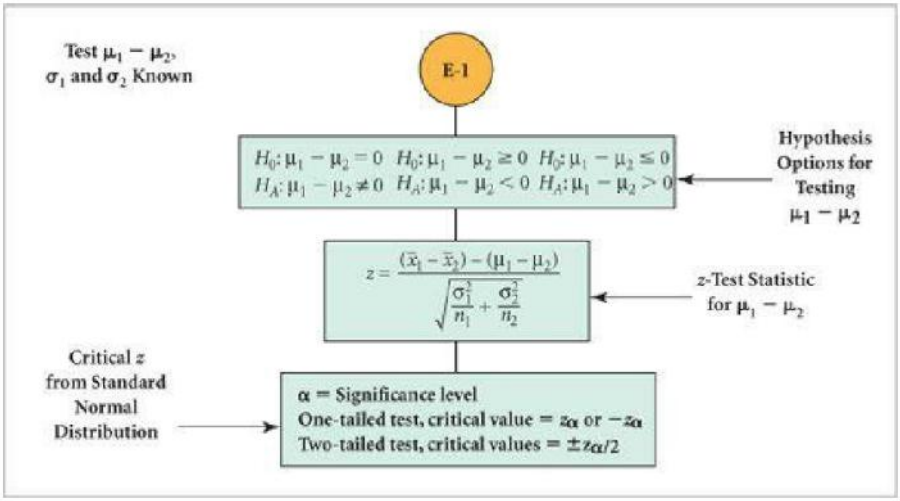

Hypothesis Test for μ, σ Known
D-1
$H_0: \mu = 20$ $H_0: \mu \le 20$ $H_0: \mu \ge 20$
$H_A: \mu \ne 20$ $H_A: \mu > 20$ $H_A: \mu < 20$
Null and Alternative Hypothesis Options for μ
$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$
z-Test Statistic
Critical z from Standard Normal Distribution
α = Significance level
One-tailed test, critical value = z_α or $-z_\alpha$
Two-tailed test, critical values = $\pm z_{\alpha/2}$

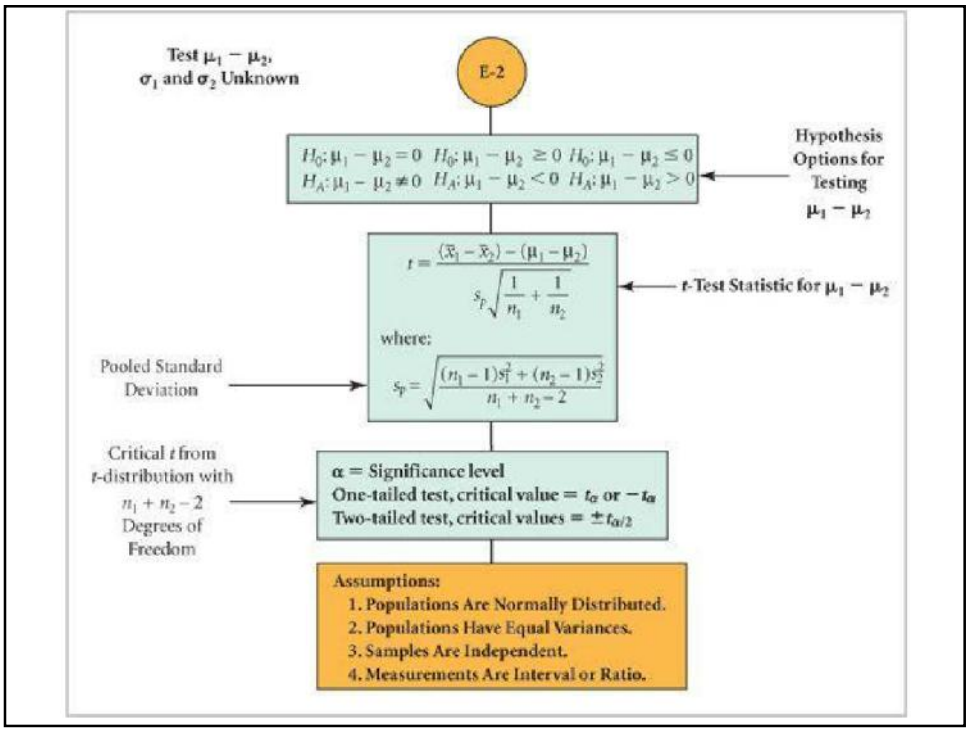

Hypothesis Test for μ, σ Unknown
D-2
$H_0: \mu = 20$ $H_0: \mu \le 20$ $H_0: \mu \ge 20$
$H_A: \mu \ne 20$ $H_A: \mu > 20$ $H_A: \mu < 20$
Null and Alternative Hypothesis Options for μ
$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$
t-Test Statistic
Critical t from t-Distribution with n − 1 Degrees of Freedom
α = Significance level
One-tailed test, critical value = t_α or $-t_\alpha$
Two-tailed test, critical values = $\pm t_{\alpha/2}$
Assumption:
Population Is Normally Distributed.

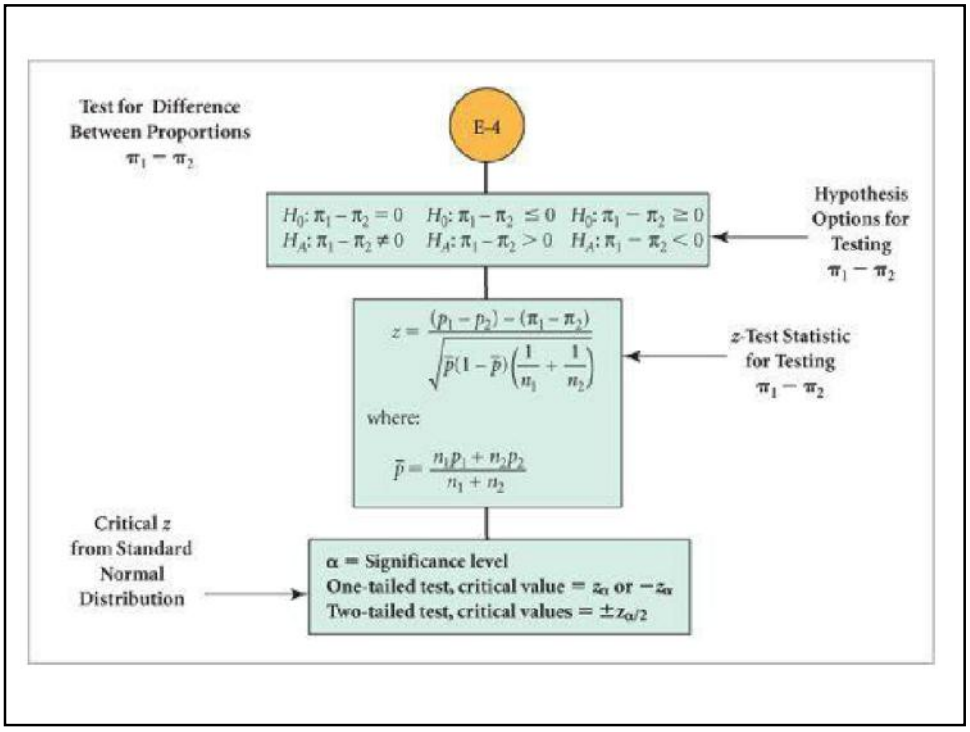

Hypothesis
Test for σ^2
D-3
$H_0: \sigma^2 = 50$ $H_0: \sigma^2 \le 50$ $H_0: \sigma^2 \ge 50$
$H_A: \sigma^2 \ne 50$ $H_A: \sigma^2 > 50$ $H_A: \sigma^2 < 50$
Null and Alternative Hypothesis Options for σ^2
$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$
χ^2 Test Statistic
α = Significance level
$df = n - 1$
One-tailed test, critical value = χ^2_α or $\chi^2_{1-\alpha}$
Two-tailed test, critical value = $\chi^2_{\alpha/2}$ and $\chi^2_{1-\alpha/2}$
Assumption:
Population Is Normally Distributed.

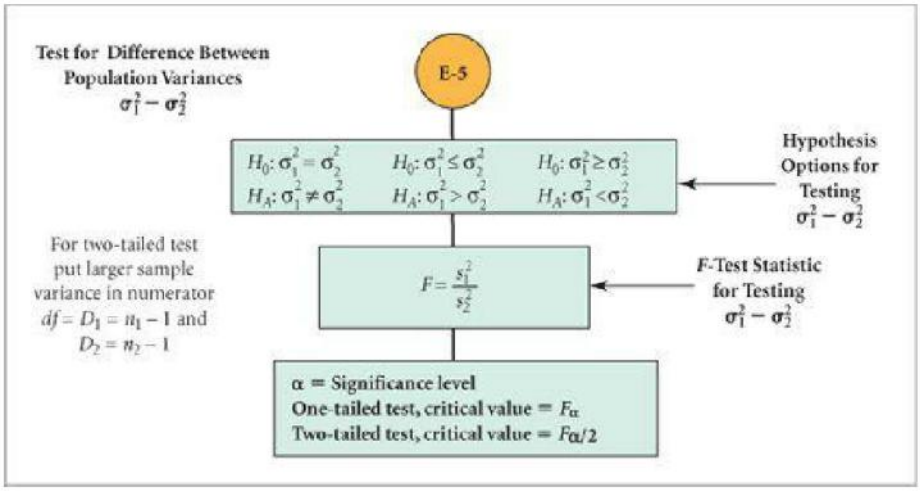

Hypothesis
Test for π
D-4
$H_0: \pi = 0.20$ $H_0: \pi \le 0.20$ $H_0: \pi \ge 0.20$
$H_A: \pi \ne 0.20$ $H_A: \pi > 0.20$ $H_A: \pi < 0.20$
Null and Alternative Hypothesis Options for π
$z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$
z-Test Statistic
Critical z from Standard Normal Distribution
α = significance level
one-tailed test, critical value = z_α or $-z_\alpha$
two-tailed test, critical values = $\pm z_{\alpha/2}$
Requirement:
$n\pi \ge 5$ and $n(1-\pi) \ge 5$

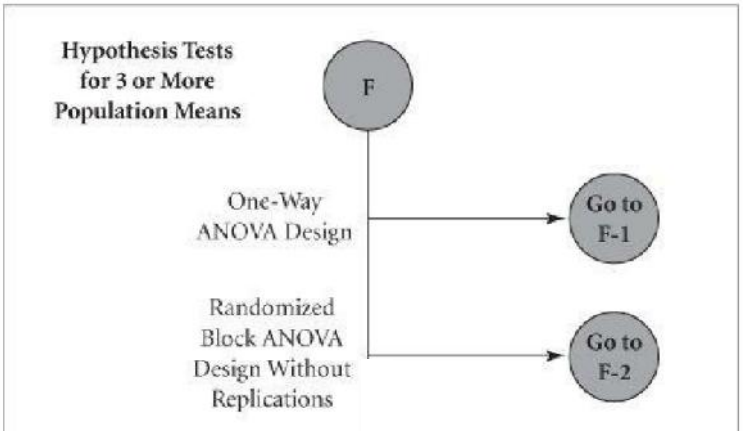

Hypothesis Tests for 2 Population Parameters
E
Population Means, Independent Samples
Population Proportions
Population Variances
Test $\mu_1 - \mu_2$
Test $\pi_1 - \pi_2$
Test $\sigma_1^2 - \sigma_2^2$
σ_1 and σ_2 known
Go to E-1
σ_1 and σ_2 unknown
Go to E-2
Go to E-3
Go to E-4


Test $\mu_1 - \mu_2$, σ_1 and σ_2 Known
E-1
$H_0: \mu_1 - \mu_2 = 0$ $H_0: \mu_1 - \mu_2 \geq 0$ $H_0: \mu_1 - \mu_2 \leq 0$
$H_A: \mu_1 - \mu_2 \neq 0$ $H_A: \mu_1 - \mu_2 < 0$ $H_A: \mu_1 - \mu_2 > 0$
Hypothesis Options for Testing $\mu_1 - \mu_2$
$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
z-Test Statistic for $\mu_1 - \mu_2$
Critical z from Standard Normal Distribution
α = Significance level
One-tailed test, critical value = z_α or $-z_\alpha$
Two-tailed test, critical values = $\pm z_{\alpha/2}$


Test $\mu_1 - \mu_2$, σ_1 and σ_2 Unknown
E-2
$H_0: \mu_1 - \mu_2 = 0$ $H_0: \mu_1 - \mu_2 \geq 0$ $H_0: \mu_1 - \mu_2 \leq 0$
$H_A: \mu_1 - \mu_2 \neq 0$ $H_A: \mu_1 - \mu_2 < 0$ $H_A: \mu_1 - \mu_2 > 0$
Hypothesis Options for Testing $\mu_1 - \mu_2$
$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$
where:
$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$
t-Test Statistic for $\mu_1 - \mu_2$
Pooled Standard Deviation
Critical t from t-distribution with $n_1 + n_2 - 2$ Degrees of Freedom
α = Significance level
One-tailed test, critical value = t_α or $-t_\alpha$
Two-tailed test, critical values = $\pm t_{\alpha/2}$
Assumptions:
1. Populations Are Normally Distributed.
2. Populations Have Equal Variances.
3. Samples Are Independent.
4. Measurements Are Interval or Ratio.


Test for Difference Between Proportions $\pi_1 - \pi_2$
E-4
$H_0: \pi_1 - \pi_2 = 0$ $H_0: \pi_1 - \pi_2 \leq 0$ $H_0: \pi_1 - \pi_2 \geq 0$
$H_A: \pi_1 - \pi_2 \neq 0$ $H_A: \pi_1 - \pi_2 > 0$ $H_A: \pi_1 - \pi_2 < 0$
Hypothesis Options for Testing $\pi_1 - \pi_2$
$z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$
where:
$\bar{p} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$
z-Test Statistic for Testing $\pi_1 - \pi_2$
Critical z from Standard Normal Distribution
α = Significance level
One-tailed test, critical value = z_α or $-z_\alpha$
Two-tailed test, critical values = $\pm z_{\alpha/2}$


Test for Difference Between Population Variances $\sigma_1^2 - \sigma_2^2$
E-5
$H_0: \sigma_1^2 = \sigma_2^2$ $H_0: \sigma_1^2 \le \sigma_2^2$ $H_0: \sigma_1^2 \ge \sigma_2^2$
$H_A: \sigma_1^2 \ne \sigma_2^2$ $H_A: \sigma_1^2 > \sigma_2^2$ $H_A: \sigma_1^2 < \sigma_2^2$
Hypothesis Options for Testing $\sigma_1^2 - \sigma_2^2$
For two-tailed test put larger sample variance in numerator $df = D_1 = n_1 - 1$ and $D_2 = n_2 - 1$
$F = \frac{s_1^2}{s_2^2}$
F-Test Statistic for Testing $\sigma_1^2 - \sigma_2^2$
α = Significance level
One-tailed test, critical value = F_α
Two-tailed test, critical value = $F_{\alpha/2}$


Hypothesis Tests for 3 or More Population Means
F
One-Way ANOVA Design
Go to F-1
Randomized Block ANOVA Design Without Replications
Go to F-2

One-Way ANOVA Design for 3 or More Population Means

F-1

Null and Alternative Hypotheses

$H_0: \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k$
H_A: At least two populations have different means.

ANOVA Table

Source of Variation	SS	df	MS	F-Ratio
Between samples	SSB	$k-1$	MSB	$\frac{MSB}{MSW}$
Within samples	SSW	n_T-k	MSW	
Total	SST	n_T-1		

Critical F from F-Distribution with $D_1 = k-1$ and $D_2 = n_T - k$ Degrees of Freedom

α = Significance level
Critical value = F_α

If null hypothesis is rejected, compare all possible pairs, $|\bar{x}_i - \bar{x}_j|$ to Tukey-Kramer Critical Range

Tukey-Kramer Critical Range

$$\text{critical range} = q_{1-\alpha}\sqrt{\frac{MSW}{2}\left(\frac{1}{n_i}+\frac{1}{n_j}\right)}$$

Assumptions:
1. **Populations Are Normally Distributed.**
2. **Populations Have Equal Variances.**
3. **Samples Are Independent.**
4. **Measurements Are Interval or Ratio.**

Randomized Block ANOVA Design for 3 or More Population Means

F-2

Primary Null and Alternative Hypotheses

$H_0: \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k$
H_A: At least two populations have different means.

Blocking Null and Alternative Hypotheses

$H_0: \mu_{b1} = \mu_{b2} = \mu_{b3} = \cdots = \mu_{bn}$ (blocking is not effective).
H_A: At least two populations have different means (blocking is effective).

ANOVA Table

Source of Variation	SS	df	MS	F-Ratio
Between blocks	$SSBL$	$b-1$	$MSBL$	$\frac{MSBL}{MSW}$
Between samples	SSB	$k-1$	MSB	$\frac{MSB}{MSW}$
Within samples	SSW	$(k-1)(b-1)$	MSW	
Total	SST	n_T-1		

Critical F from F-Distribution

α = Significance level
Blocking critical value = F_α
$df = D_1 = b - 1$ and $D_2 = (k-1)(b-1)$
Primary critical value = F_α, $df = D_1 = k - 1$ and $D_2 = (k-1)(b-1)$

Assumptions:

1. Populations Are Normally Distributed.
2. Populations Have Equal Variances.
3. Observations Within Samples Are Independent.
4. Measurements Are Interval or Ratio.

If null is rejected, compare all $|\bar{x}_i - \bar{x}_j|$ to

Fisher's $LSD = t_{\alpha/2}\sqrt{MSW}\sqrt{\frac{2}{b}}$