

Answers

15.1

- a. $b_1 = 4.14$. This implies that, holding x_2 constant and increasing x_1 by one unit, the average y is estimated to increase by 4.14 units. $b_2 = 8.72$. This implies that, holding x_1 constant and increasing x_2 by one unit, the average y is estimated to increase by 8.72 units.
- b. $\hat{y} = 12.67 + 4.14(4) + 8.72(9) = 107.71$

15.2

- a. $b_1 = -412$. This implies that, holding the other independent variables constant and increasing the local unemployment rate by one percent, the average weekly sales is estimated to decrease by 412 dollars. $b_2 = 818$. This implies that, holding the other independent variables constant and increasing the weekly average high temperature by one degree, the average weekly sales is estimated to increase by 818 dollars. $b_3 = -93$. This implies that, holding the other independent variables constant and increasing the number of local activities by one, the average weekly sales is estimated to decrease by 93 dollars. $b_4 = -71$. This implies that, holding the other independent variables constant and increasing the average gasoline price by one dollar, the average weekly sales is estimated to decrease by 71 dollars.
- b. $\hat{y} = 22,167 - 412(5.7) + 818(61) - 93(14) - 71(1.39) = 68,315.91$

15.3

- a. $\hat{y} = 87.7897 - 0.9705x_1 + 0.0023x_2 - 8.7233x_3$
- b. $F = 5.3276 > F_{0.05} = 3.0725$ (Using Excel's FINV) Also, $p\text{-value} = .00689 < \text{any reasonable alpha}$. Therefore, reject $H_0: \beta_1 = \beta_2 = \beta_3 = 0$. At least some part of the model is statistically significant.
- c. $R^2 = \frac{SSR}{SST} = \frac{16646.091}{38517.76} = 0.432$
- d. x_1 ($p\text{-value} = 0.1126 > \alpha = 0.05 \rightarrow$ fail to reject $H_0: \beta_1 = 0$) and x_3 ($p\text{-value} = 0.2576 > \alpha = 0.05 \rightarrow$ fail to reject $H_0: \beta_3 = 0$) are not significant.
- e. $b_2 = 0.0023 \rightarrow \hat{y}$ increases 0.0023 for each one unit increase of x_2 .
 $b_3 = -8.7233 \rightarrow \hat{y}$ decreases 8.7233 for each one unit increase of x_3 .
- f. The confidence intervals for β_1 and β_3 contain 0. This indicates that x_1 and x_3 are not statistically significantly different than 0 in this model.

15.4

The largest correlation that exists among these independent variables is 0.504.
 From the previous problem, $n = 25$, so:

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{0.502}{\sqrt{\frac{1-0.502^2}{25-2}}} = 2.78$$

This is significant at the .05 level but not at the .01 level. This is the only potential multicollinearity problem.

15.5

- a. Minitab output:
 Regression Analysis: y_i versus x_1, x_2
 The regression equation is
 $y_i = 5.05 - 0.051 x_1 + 0.888 x_2$

Predictor	Coef	SE Coef	T	P	VIF
Constant	5.045	8.698	0.58	0.580	
x_1	-0.0513	0.2413	-0.21	0.838	1.1
x_2	0.8880	0.1475	6.02	0.001	1.1

S = 6.82197 R-Sq = 84.5% R-Sq(adj) = 80.1%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	1775.12	887.56	19.07	0.001
Residual Error	7	325.78	46.54		
Total	9	2100.90			

The estimated regression equation is $\hat{y} = 5.05 - 0.0513x_1 + 0.888x_2$

b. Minitab output:

Correlations: yi, x1, x2

	yi	x1
x1	0.206	
x2	0.919	0.257

Cell Contents: Pearson correlation

Using the steps from Chapter 14:

Step 1: Define the parameter of interest.

The parameter of interest is the population correlation coefficient between y and x_1 , ρ ;

Step 2: Formulate the appropriate null and alternative hypotheses.

$$H_0: \rho = 0 \quad H_A: \rho \neq 0$$

Step 3: Specify the level of significance.

$$\alpha = 0.05$$

Step 4: Compute the test statistic.

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{0.206}{\sqrt{\frac{1-0.206^2}{10-2}}} = 0.5954$$

Step 5: Construct the rejection region.

The degrees of freedom are $n - 2 = 8$; the critical values are ± 2.306 .

Step 6: Reach a decision.

Since $-2.306 < t = 0.5954 < 2.306$; we fail to reject H_0 .

Step 7: Draw a conclusion.

The correlation with the dependent variable is not significant.

c. Step 1: The parameters of interest are the population coefficients of x_1 and x_2 ,

Step 2: $H_0: \beta_1 = \beta_2 = 0$, H_A : at least one $\beta_i \neq 0$

Step 3: $\alpha = 0.05$

$$\text{Step 4: } F = \frac{\frac{SSR}{k}}{\frac{SSE}{n-k-1}} = \frac{\frac{1775.12}{2}}{\frac{325.78}{10-2-1}} = 19.07$$

Step 5: The numerator degrees of freedom are $k = 2$ and the denominator degrees of freedom are $n - k - 1 = 10 - 2 - 1 = 7$; the critical value = 4.737.

Step 6: Since $F = 19.07 > 4.737$, reject H_0 ;

Step 7: The overall model is significant.

d. Minitab output:

Predictor	Coef	SE Coef	T	P	VIF
Constant	5.045	8.698	0.58	0.580	
x1	-0.0513	0.2413	-0.21	0.838	1.1
x2	0.8880	0.1475	6.02	0.001	1.1

S = 6.82197 R-Sq = 84.5% R-Sq(adj) = 80.1%

A $VIF < 5$ for a given independent variable indicates that this independent variable is not correlated with the remaining independent variables in the model. Both x_1 and x_2 have VIFs equal to 1.1. Therefore, neither of the independent variable is correlated with the other independent variable. Multicollinearity does not exist between the two independent variables.

15.14

- $\hat{y} = 24.1 + 5.8x_1 + 7.9(1) = (24.1 + 7.9) + 5.8x_1 = 32 + 5.8x_1$
- $\hat{y} = 24.1 + 5.8(10) + 7.9(1) = 32 + 5.8(10) = 90$
- $\hat{y} = 24.1 + 5.8x_1 + 7.9(0) = 24.1 + 5.8x_1$
- $\hat{y} = 24.1 + 5.8(30) + 7.9(0) = 198.1$

15.15

- Since the apartment is in the town center, $x_2 = 1$ which implies $\hat{y} = 145 + 1.2(1500) + 300(1) = 2245$.
- Since the apartment is not in the town center, $x_2 = 0$ which implies $\hat{y} = 145 + 1.2(1500) + 300(0) = 1945$.
- The difference between the answers in part a. and b. ($2245 - 1945 = 300$) equals the value of b_2 . Therefore, b_2 indicates the average premium paid for living in the city's town center.

15.18

- Minitab output:

Regression Analysis: Price versus Sq Footage, Type

The regression equation is
 Price = 69631 + 90.4 Sq Footage - 3630 Type

Predictor	Coef	SE Coef	T	P
Constant	69631	53986	1.29	0.214
Sq Footage	90.37	29.53	3.06	0.007
Type	-3630	15891	-0.23	0.822

S = 32186.4 R-Sq = 40.6% R-Sq(adj) = 33.7%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	12062286443	6031143222	5.82	0.012
Residual Error	17	17611399057	1035964650		
Total	19	29673685500			

$$\hat{y} = 69631 + 90.4x_1 - 3630x_2$$

- β_1 = the average change in the selling price of a residence (for both condominiums and single family residences) when the square footage of the residence is increased by one square foot. β_2 = the difference in the average selling price of a residence for a specified square footage between condominiums and single family residences.
- The equation that describes the relationship between the selling price and the square footage for condominiums is $\hat{y} = 69631 + 90.4x_1 - 3630(1) = 66001 + 90.4x_1$, and for single family homes $\hat{y} = 69631 + 90.4x_1 - 3630(0) = 69631 + 90.4x_1$.
- Since $-2.1098 < t = -0.23 < 2.1098$, we fail to reject H_0 . There is sufficient evidence to conclude that the relationship between the selling price and the square footage is not different between condominiums and single family homes.