QM353: Business Statistics

Chapter 4

11.4. Given the following null and alternative hypotheses
H_0: $\sigma^2 = 100$
H_A: $\sigma^2 \neq 100$
a. Test when $n = 27$, $s = 9$, and $\alpha = 0.10$. Be certain to state the decision rule.
b. Test when $n = 17$, $s = 6$, and $\alpha = 0.05$. Be certain to state the decision rule.

11.6. Suppose a random sample of 22 items produces a sample standard deviation of 16.
a. Use the sample results to develop a 90% confidence interval estimate for the population variance.
b. Use the sample results to develop a 95% confidence interval estimate for the population variance.

11.8. Examine the sample obtained from a normally distributed population:

5.2	10.4	5.1	2.1	4.8	15.5	10.2
8.7	2.8	4.9	4.7	13.4	15.6	14.5

a. Calculate the variance.
b. Calculate the probability that a randomly chosen sample would produce a sample variance at least as large as that produced in part a if the population variance was equal to 20.
c. What is the statistical term used to describe the probability calculated in part b?
d. Conduct a hypothesis test to determine if the population variance is larger than 15.3. Use a significance level equal to 0.05.

11.12. Airlines face the challenging task of keeping their planes on schedule. One key measure is the number of minutes a plane deviates from the targeted arrival time. Ideally, the measure for each arrival will be zero minutes, indicating that the plane arrived exactly on time. However, experience indicates that even under the best of circumstances there will be inherent variability. Suppose one major airline has set standards that require the planes to arrive, on average, on time, with a standard deviation not to exceed two minutes. To determine whether these standards are being met, each month the airline selects a random sample of 12 airplane arrivals and determines the number of minutes early or late the flight is. For last month, the times, rounded to the nearest minute, are

3	−7	4	2	−2	5	11	−3	4	6	−4	1

a. State the appropriate null and alternative hypothesis for testing the standard regarding the mean value. Test the hypothesis using a significance level equal to 0.05. What assumption will be required?
b. State the appropriate null and alternative hypotheses regarding the standard deviation. Use the sample data to conduct the hypothesis test with $\alpha = 0.05$.
c. Discuss the results of both tests. What should the airline conclude regarding its arrival standards? What factors could influence the arrival times of flights?

11.20. Given the following null and alternative hypotheses

$$H_0: \sigma_1^2 = \sigma_2^2$$
$$H_A: \sigma_1^2 \neq \sigma_2^2$$

and the following sample information

Sample 1	Sample 2
$n_1 = 11$	$n_2 = 21$
$s_1 = 15$	$s_2 = 33$

a. If $\alpha = 0.02$, state the decision rule for the hypothesis.
b. Test the hypothesis and indicate whether the null hypothesis should be rejected.

11.22. Given the following null and alternative hypotheses

$$H_0\colon \sigma_1^2 \le \sigma_2^2$$
$$H_A\colon \sigma_1^2 > \sigma_2^2$$

and the following sample information

Sample 1	Sample 2
$n_1 = 21$	$n_2 = 12$
$s_1^2 = 345.7$	$s_2^2 = 745.2$

a. If α 0.01, state the decision rule for the hypothesis. (Be careful to pay attention to the alternative hypothesis to construct this decision rule.)
b. Test the hypothesis and indicate whether the null hypothesis should be rejected.

11.23. Consider the following two independently chosen samples:

Sample 1	Sample 2
12.1	10.5
13.4	9.5
11.7	8.2
10.7	7.8
14.0	11.1

Use a significance level of 0.05 for testing the hypothesis that $\sigma_1^2 \le \sigma_2^2$.

Answers

11.4

a.

H_0: $\sigma^2 = 100$ H_A: $\sigma^2 \ne 100$

This is a two-tailed test.

The random sample consists of n = 27 observations. The sample variance is $s^2 = 9^2 = 81$. The test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(27-1)81}{100} = 21.06$$

Because $\chi^2 = 21.06 < \chi^2_{0.05} = 38.8851$ and because $\chi^2 = 21.06 > \chi^2_{0.95} = 15.3792$ do not reject the null hypothesis based on these sample data.

Based on the sample data and the hypothesis test conducted we do not reject the null hypothesis at the α = 0.10 level of significance and we conclude the population variance is not different from 100.

b.

H_0: $\sigma^2 = 100$ H_A: $\sigma^2 \neq 100$

This is a two-tailed test.

The random sample consists of n = 17 observations. The sample variance is $s^2 = 6^2 = 36$. The test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(17-1)36}{100} = 5.76$$

.

Because $\chi^2 = 5.76 < \chi^2_{0.975} = 6.9077$ we reject the null hypothesis.

Based on the sample data and the hypothesis test conducted we do reject the null hypothesis at the α = 0.05 level of significance and conclude the population variance is different from 100.

11.6

a.

The confidence interval estimate for the population variance, σ^2 is computed using Equation 11-2 shown below:

$$\frac{(n-1)s^2}{\chi^2_U} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_L}$$

For a 90% confidence interval we find the following values for χ^2_U and χ^2_L with n-1 = 22-1 = 21 degrees of freedom:

$\chi^2_{0.95} = 11.5913$ and $\chi^2_{0.05} = 32.6706$

The confidence interval is calculated using Equation 11-2:

$$\frac{(22-1)16^2}{32.6706} \leq \sigma^2 \leq \frac{(22-1)16^2}{11.5913} = 164.55 \leq \sigma^2 \leq 463.80$$

b.

The confidence interval estimate for the population variance, σ^2 is computed using Equation 11-2. For a 95% confidence interval we find the following values for χ^2_U and χ^2_L with n-1 = 22-1 = 21 degrees of freedom:

$\chi^2_{0.975} = 10.2829$ and $\chi^2_{0.025} = 35.4789$

The confidence interval is calculated using Equation 11-2:

$$\frac{(22-1)16^2}{35.4789} \leq \sigma^2 \leq \frac{(22-1)16^2}{10.2829} = 151.53 \leq \sigma^2 \leq 522.81$$

11.8

a. $s^2 = \dfrac{\sum (x-\bar{x})^2}{n-1} = \dfrac{302.86}{14-1} = 23.297.$

b. Since $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{(14-1)23.297}{20} = 15.14$, $P(s^2 \geq 23.297) = P(\chi^2 \geq 15.14) \approx 0.30$. Therefore, p-value ≈ 0.30.

c. This is the observed probability of rejecting the null hypothesis when the null hypothesis is true: the p-value.

d. Using the seven step procedure outlined in the chapter:

Step 1: The parameter of interest is the population variance, σ^2

Step 2: H_O: $\sigma^2 \leq 15.3$ H_A: $\sigma^2 > 15.3$.

Step 3: α = 0.05

Step 4: Since $¡^2 = \frac{(n-1)s^2}{\pm^2} = \frac{(14-1)23.297}{15.3} = 19.79$, P($¡^2$ ≥ 19.79) ≈ 0.10. Therefore, p-value ≈ 0.10.
Step 5: $\alpha = 0.05 <$ p-value ≈ 0.10.,
Step 6: Fail to reject H_O,
Step 7: Conclude that there is not enough evidence to conclude that the variance is larger than 15.3.

11.12

a. H_o: $\mu = 0$
H_a: $\mu \neq 0$
Using Excel's AVERAGE and STDEV functions
$\bar{x} = 1.6667$ s = 4.9787
$t = (1.6667 - 0)/(4.9787/\sqrt{12}) = 1.1597$
$t_{.05/2} = \pm 2.2010$
Since t = 1.1597 < 2.2010 do not reject H_o and conclude that the average arrival time is on time.
Because this is a t-distribution you must assume that the underlying population is normally distributed.

b. H_o: $\sigma^2 \leq 4$
H_a: $\sigma^2 > 4$
$\chi^2 = [(12-1)(4.9787)^2]/4 = 68.1655$
Decision Rule:
If $\chi^2 > 19.6752$, reject H_o, otherwise do not reject H_o
Since 68.1655 > 19.6752 do reject Ho and conclude that the population variance is greater than 4

c. From part a and b airlines should conclude that on the average the planes arrive on time but with variance greater than 4, Factors that could influence arrival times are weather, availability of planes and/or crew, other flight traffic; and capacity of airports.

11.20

a. If the calculated F > 4.405, reject H_0, otherwise do not reject H_0
b. $F = 33^2/15^2 = 4.84$
Since 4.84 > 4.405 reject H_0

11.22

a. Using Appendix H: If the calculated F >3.858, reject H_0, otherwise do not reject H_0
b. F = 345.7/745.2 = 0.46390
Since 0.46390 < 3.858 do not reject H_0

11.23

Using the seven step procedure from the chapter:

Step 1: $\pm^2$,

Step 2: H_O: $\pm_1^2 \leq \pm_2^2$, H_A: $\pm_1^2 > \pm_2^2$,
Step 3: $\alpha = 0.05$,
Step 4: The critical value is obtained from the F-distribution. F = 6.388. Reject H_O if F > 6.388.

Step 5: $s_1^2 = \frac{7.028}{4} = 1.757$ $s_2^2 = \frac{\sum(x-\bar{x})^2}{n-1} = \frac{8.108}{4} = 2.027$. The test statistic is F = $\frac{s_2^2}{s_1^2} = \frac{2.027}{1.757} = 1.154$,

Step 6: Since F = 1.154 < 6.388 = $F_{0.05}$, fail to reject H_O.
Step 7: Based on these sample data, there is not sufficient evidence to conclude that population one's variance is larger than population two's variance.