QM353: Business Statistics

Chapter 3

9.2. For the following hypothesis test:

$$H_0: \mu = 200$$
$$H_A: \mu \neq 200$$
$$\alpha = 0.01$$

with $n = 64$, $\sigma = 9$, and $\bar{x} = 196.5$, state
a. the decision rule in terms of the critical value of the test statistic
b. the calculated value of the test statistic
c. the conclusion

9.4. For the following hypothesis test:

$$H_0: \mu \leq 45$$
$$H_A: \mu > 45$$
$$\alpha = 0.02$$

with $n = 80$, $\sigma = 9$, and $\bar{x} = 47.1$, state
a. the decision rule in terms of the critical value of the test statistic
b. the calculated value of the test statistic
c. the appropriate *p*-value
d. the conclusion

9.8. For each of the following pairs of hypotheses, determine whether each pair represents valid hypotheses for a hypothesis test. Explain reasons for any pair that is indicated to be invalid.
a. H_0: $\mu = 15$, H_A: $\mu > 15$
b. H_0: $\mu = 20$, H_A: $\mu \neq 20$
c. H_0: $\mu < 30$, H_A: $\mu > 30$
d. H_0: $\mu \leq 40$, H_A: $\mu \geq 40$
e. H_0: ≤ 45, H_A: > 45
f. H_0: $\mu \leq 50$, H_A: $\mu > 55$

9.12. A mail-order business prides itself in its ability to fill customers' orders in six calendar days or less on the average. Periodically, the operations manager selects a random sample of customer orders and determines the number of days required to fill the orders. Based on this sample information, he decides if the desired standard is not being met. He will assume that the average number of days to fill customers' orders is six or less unless the data suggest strongly otherwise.
a. Establish the appropriate null and alternative hypotheses.
b. On one occasion where a sample of 40 customers was selected, the average number of days was 6.65, with a sample standard deviation of 1.5 days. Can the operations manager conclude that his mail-order business is achieving its goal? Use a significance level of 0.025 to answer this question.
c. Calculate the *p*-value for this test. Conduct the test using this *p*-value.
d. The operations manager wishes to monitor the efficiency of his mail-order service often. Therefore, he does not wish to repeatedly calculate *t*-values to conduct the hypothesis tests. Obtain the critical value, $\bar{x}_{\sim}$, so that the manager can simply compare the sample mean to this value to conduct the test. Use $\bar{x}$ as the test statistic to conduct the test.

9.14. The director of a state agency believes that the average starting salary for clerical employees in the state is less than $30,000 per year. To test her hypothesis, she has collected a simple random sample of 100 starting clerical salaries from across the state and found that the sample mean is $29,750.

a. State the appropriate null and alternative hypotheses.
b. Assuming the population standard deviation is known to be $2,500 and the significance level for the test is to be 0.05, what is the critical value (stated in dollars)?
c. Referring to your answer in part b, what conclusion should be reached with respect to the null hypothesis?
d. Referring to your answer in part c, which of the two statistical errors might have been made in this case? Explain.

9.24. For the following hypothesis test:

$$H_0: \pi = 0.40$$
$$H_A: \pi \neq 0.40$$
$$\alpha = 0.01$$

with $n = 64$ and $p = 0.42$, state
a. the decision rule in terms of the critical value of the test statistic
b. the calculated value of the test statistic
c. the conclusion

9.28. A test of hypothesis has the following hypotheses:

$$H_0: \pi \leq 0.45$$
$$H_A: \pi > 0.45$$

For a sample size of 30, and a sample proportion of 0.55,
a. For an $\alpha = 0.025$, determine the critical value.
b. Calculate the numerical value of the test statistic.
c. State the test's conclusion.
d. Determine the *p*-value.

9.30. A major issue facing many states is whether to legalize casino gambling. Suppose the governor of one state believes that more than 55% of the state's registered voters would favor some form of legal casino gambling. However, before backing a proposal to allow such gambling, the governor has instructed his aides to conduct a statistical test on the issue. To do this, the aides have hired a consulting firm to survey a simple random sample of 300 voters in the state. Of these 300 voters, 175 actually favored legalized gambling.
a. State the appropriate null and alternative hypotheses.
b. Assuming that a significance level of 0.05 is used, what conclusion should the governor reach based on these sample data? Discuss.

9.32. A *Washington Post*-ABC News poll found that 72% of people are concerned about the possibility that their personal records could be stolen over the Internet. If a random sample of 300 college students at a Midwestern university were taken and 228 of them were concerned about the possibility that their personal records could be stolen over the Internet, could you conclude at the 0.025 level of significance that a higher proportion of the university's college students are concerned about Internet theft than the public at large? Report the *p*-value for this test.

10.3. Construct a 95% confidence interval estimate for the difference between two population means based on the following information:

Population 1	Population 2
$\bar{x}_1 = 355$	$\bar{x}_2 = 320$
$\sigma_1 = 34$	$\sigma_2 = 40$
$n_1 = 50$	$n_2 = 80$

10.4. Construct a 90% confidence interval estimate for the difference between two population means given the following sample data selected from two normally distributed populations with equal variances:

Sample 1			Sample 2		
29	25	31	42	39	38
35	35	37	42	40	43
21	29	34	46	39	35

10. 7. A pet food producer manufactures and then fills 25-pound bags of dog food on two different production lines located in separate cities. In an effort to determine whether differences exist between the average fill rates for the two lines, a random sample of 19 bags from line 1 and a random sample of 23 bags from line 2 were recently selected. Each bag's weight was measured and the following summary measures from the samples are reported:

	Production Line 1	Production Line 2
Sample Size, n	19	23
Sample Mean, $\bar{x}$	24.96	25.01
Sample Standard Deviation, s	0.07	0.08

Management believes that the fill rates of the two lines are normally distributed with equal variances.
a. Calculate the point estimate for the difference between the population means of the two lines.
b. Develop a 95% confidence interval estimate of the true mean difference between the two lines.
c. Based on the 95% confidence interval estimate calculated in part b, what can the managers of the production lines conclude about the differences between the average fill rates for the two lines?

10.18. Given the following null and alternative hypotheses
H_0: $\mu_1 \geq \mu_2$
H_A: $\mu_1 < \mu_2$
together with the following sample information

Sample 1	Sample 2
$n_1 = 14$	$n_2 = 18$
$\bar{x}_1 = 565$	$\bar{x}_2 = 578$
$s_1 = 28.9$	$s_2 = 26.3$

a. Assuming that the populations are normally distributed with equal variances, test at the 0.10 level of significance whether you would reject the null hypothesis based on the sample information. Use the test statistic approach.
b. Assuming that the populations are normally distributed with equal variances, test at the 0.05 level of significance whether you would reject the null hypothesis based on the sample information. Use the test statistic approach.

10.19. Given the following null and alternative hypotheses, conduct a hypothesis test using an alpha equal to 0.05. (*Note*: The population standard deviations are assumed to be known.)
H_0: $\mu_1 \leq \mu_2$
H_A: $\mu_1 > \mu_2$
The sample means for the two populations are shown as follows:

$\bar{x}_1 = 144$	$\bar{x}_2 = 129$
$\sigma_1 = 11$	$\sigma_2 = 16$
$n_1 = 40$	$n_2 = 50$

10.26. Descent, Inc., produces a variety of climbing and mountaineering equipment. One of its products is a traditional three-strand climbing rope. An important characteristic of any climbing rope is its tensile strength. Descent produces the three-strand rope on two separate production lines: one in Bozeman and the other in Challis. The Bozeman line has recently installed new production equipment. Descent regularly tests the tensile strength of its ropes by randomly selecting ropes from production and subjecting them to various tests. The most recent random sample of ropes, taken after the new equipment was installed at the Bozeman plant, revealed the following:

Bozeman	Challis
$\bar{x}_1 = 7{,}200$ lb	$\bar{x}_2 = 7{,}087$ lb
$s_1 = 425$	$s_2 = 415$
$n_1 = 25$	$n_2 = 20$

Descent's production managers are willing to assume that the population of tensile strengths for each plant is approximately normally distributed with equal variances. Based on the sample results, can Descent's managers conclude that there is a difference between the mean tensile strengths of ropes produced in Bozeman and Challis? Conduct the appropriate hypothesis test at the 0.05 level of significance.

10.55. Independent random samples of size 50 and 75 are selected. The sampling results in 35 and 35 successes, respectively. Test the following hypotheses:

a. H_0: $\pi_1 - \pi_2 = 0$ vs. H_A: $\pi_1 - \pi_2 \neq 0$. Use $\alpha = 0.05$.
b. H_0: $\pi_1 - \pi_2 \geq 0$ vs. H_A: $\pi_1 - \pi_2 < 0$. Use $\alpha = 0.05$.
c. H_0: $\pi_1 - \pi_2 \leq 0$ vs. H_A: $\pi_1 - \pi_2 < 0$. Use $\alpha = 0.025$.
d. H_0: $\pi_1 - \pi_2 = 0.05$ vs. H_A: $\pi_1 - \pi_2 \neq 0$. Use $\alpha = 0.02$.

10.56. Suppose a random sample of 100 U.S. companies taken in 2005 showed that 21 offered high deductible health insurance plans to their workers. A separate random sample of 120 firms taken in 2006 showed that 30 offered high-deductible health insurance plans to their workers. Based on the sample results, can you conclude that there is a higher proportion of U.S. companies offering high deductible health insurance plans to their workers in 2006 than in 2005? Conduct your hypothesis test at a level of significance $\alpha = 0.05$.

Answers:

9.2.

a. This is a two-tailed test of the population mean when the population standard deviation is known. Therefore, the decision rule is: reject the null hypothesis if the calculated value of the test statistic, z, is greater than 2.575 or less than – 2.575. Otherwise, do not reject.

b. $z = (196.5-200)/(9/\sqrt{64}) = -3.111$

c. Because the calculated value of the test statistic, z = -3.111, is less than -2.575, reject the null hypothesis and conclude that the population mean is not equal to 200.

9.4.

a. This is a one-tailed test of the population mean with σ known. Therefore, the decision rule is: reject the null hypothesis if the calculated value of the test statistic, *z*, is greater than the critical value of the test statistic, 2.05. Otherwise, do not reject.
b. $z = (47.1-45)/(9/\sqrt{80}) = 2.087$.
c. From the standard normal table, p-value = 0.5 - 0.4817 = 0.0183
d. Because the computed value of $z = 2.087$ is greater than 2.05, reject the null hypothesis and conclude the mean is greater than 45. Also because the p-value is less than 0.02

9.8.

a. Invalid. The null hypothesis contains an equality and the alternative hypothesis must include all population values not covered by the null hypothesis.
b. Valid
c. Invalid. The null hypothesis should contain an equality sign.
d. Invalid. The alternative hypothesis cannot contain an equality sign.
e. Invalid. Hypotheses concern population parameters and not sample statistics such as $\bar{x}$.
f. Invalid. The numerical value specified in the null and alternative hypotheses must be the same.

9.12.

a. H_o: $\mu \le 6$ days

H_a: $\mu > 6$ days

b. $$t = \frac{\bar{x} - «}{\frac{s}{\sqrt{n}}} = \frac{6.65 - 6.00}{\frac{1.5}{\sqrt{40}}} = \frac{.65}{.2372} = 2.7406$$

Using Excel's TINV option, $t_{.025} = 2.023$

Since 2.7406 > 2.023 reject H_o and conclude that the mail-order business is not achieving its goal.

c. Using Excel's TDIST option, the p-value = P(t > 2.74) = .0046. Since .0046 < .025, reject the null hypothesis.

d. Must assume that the population standard deviation is 1.5 and that the sample size is set at 40.

$\bar{x}_\alpha = 6.00 + 1.96(1.5/\sqrt{40}) = 6.4649$

If $\bar{x} > 6.4649$, reject the null hypothesis

Otherwise, do not reject.

9.14.

a. H_0: $\mu \geq 30{,}000$

H_A: $\mu < 30{,}000$

b. For alpha = .05 and a one tailed , lower tail test, the critical value is z = -1.645. Solving for the critical x-bar: -1.645 = (x-bar – 30,000)/250, x-bar = $30,411.25

c. Since the sample value of $29,750 is less than the critical value of $30,411.25, do not reject the null hypothesis.

d. A Type II error could have been made since the null hypothesis was not rejected.

9.24.

a. This is a two-tailed test of the population proportion. The decision rule is: reject the null hypothesis if the calculated value of the test statistic, z, is greater than 2.575 or less than – 2.575. Otherwise, do not reject.

b. z = (0.42-0.40)/($\sqrt{0.40*(1-0.40)/64}$) = 0.3266

c. Because the calculated value of the test statistic, z = 0.3266, is neither greater than 2.575 nor less than -2.575, do not reject the null hypothesis and conclude that the population proportion is not different from 0.40.

9.28.

a. $z_{\alpha} = 1.96$

b. $$z = \frac{p - ®}{\sqrt{\frac{®(1-®)}{n}}} = \frac{0.55 - 0.45}{\sqrt{\frac{0.45(.55)}{30}}} = 1.101$$

c. $z = 1.101 < z_{\alpha} = 1.96$, therefore do not reject H_O

d. p-value = probability(z >1.101) = 0.5 – 0.3643 = 0.1357

9.30.

a. H_o: $\pi \leq 0.55$
H_a: $\pi > 0.55$

b. p= 175/300 = .5833

$$z = \frac{p - ®}{\sqrt{\frac{®(1-®)}{n}}} = \frac{.5833 - .55}{\sqrt{\frac{.55(1-.55)}{300}}} = 1.1594$$

Since z = 1.1594 < 1.645, do not reject the null hypothesis.
The sample data do not provide sufficient evidence to conclude that more than 55 percent of the population favor legalized gambling.

9.32.

x =228

p = x/n = 228/300 = 0.76

$$z = \frac{0.76 - 0.72}{\sqrt{\frac{0.72*(1-0.72)}{300}}} = 1.543$$

Because z = 1.543 is less than 1.96, do not reject Ho.
The proportion of college students at this university who are concerned about Internet theft is not greater than the general public.

p-value = P(z > 1.543)
= 0.50 – 0.4382 = 0.0618

10.3.

The standard error of the sampling distribution.

$$\sqrt{\frac{34^2}{50} + \frac{40^2}{80}}$$

Because the population standard deviations are assumed to be known, the critical value is a z-value from the standard normal distribution. The critical value is z = 1.96.
Given that the population standard deviations are know, the confidence interval estimate is developed using:

Then, the 95 percent confidence interval estimate is:

$$35 \pm 12.87$$

22.13 ≤ ≤ 47.87

10.4.

$$s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(9-1)5.24^2+(9-1)3.2^2}{9+9-2}} = 4.34$$

The critical value comes from a t-distribution with $df = n_1 + n_2 - 2$. The degrees of freedom is 9 + 9 – 2 = 16.

The critical value from the t-distribution table is 1.7459.

When the population variances are assumed to be equal, the confidence interval estimate is computed using:

Then

$$-9.77 \pm 1.7459(4.34)\sqrt{\frac{1}{9}+\frac{1}{9}}$$

-9.77 ± 3.57

-13.34 ≤ ≤ -6.2

10.7.

a. Point estimate for the difference between the two population means is

$\bar{x}_1 - \bar{x}_2$ = 24.96 – 25.01 = -0.05

b. Develop a confidence interval using equation 10-4.

$$(\bar{x}_1 - \bar{x}_2) \pm t s_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(19-1)0.07^2+(23-1)0.08^2}{19+23-2}} = 0.0757$$

Then the interval estimate is

$$-0.05 \pm 2.0211(0.0757)\sqrt{\frac{1}{19}+\frac{1}{23}} = -0.05 \pm 0.0474 =$$

$-0.0974 \le (\mu_1 - \mu_2) \le -0.0026$

c. Since the interval does not contain zero, the managers can conclude the two lines do not fill bags with equal average amounts. However, the difference is at most about 0.1 lbs.

10.18.
a.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

The pooled standard deviation is

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$s_p = \sqrt{\frac{(14-1)28.9^2 + (18-1)26.3^2}{14+18-2}} = 27.457$$

Then the t-statistic is

$$t = \frac{(565-578)-(0)}{27.457\sqrt{\frac{1}{14} + \frac{1}{18}}} = -1.329$$

Because the calculated value of t = -1.329 is less than the critical value of t=-1.3104, reject the null hypothesis.
Based on these sample data, at the α = 0.10 level of significance there is sufficient evidence to conclude that the mean for population 1 is less than the mean for population 2.
b.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

The pooled standard deviation is

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$s_p = \sqrt{\frac{(14-1)28.9^2 + (18-1)26.3^2}{14+18-2}} = 27.457$$

Then the t-statistic is

$$t = \frac{(565-578)-(0)}{27.457\sqrt{\frac{1}{14} + \frac{1}{18}}} = -1.329$$

Otherwise do not reject the null hypothesis.
Because the calculated value of t = -1.329 is not less than the critical value of t = -1.6973, do not reject the null hypothesis.
Based on these sample data, at the α = 0.05 level of significance there is not sufficient evidence to conclude that the mean for population 1 is less than the mean for population 2.

10.19.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Substituting we get:

$$z = \frac{(144-129)-0}{\sqrt{\frac{11^2}{40}+\frac{16^2}{50}}} = 5.26$$

Since z = 5.26 > 1.645, we reject the null hypothesis.
Based on the sample data we conclude that the mean for population 1 exceeds the mean for population 2.

10.26.

$H_o : \mu_1 = \mu_2$

$H_A : \mu_1 \neq \mu_2$

The test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

The pooled standard deviation is

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$$s_p = \sqrt{\frac{(25-1)425^2 + (20-1)415^2}{25+20-2}} = 420.611$$

Then the t-statistic is

$$t = \frac{(7200-7087)-(0)}{420.611\sqrt{\frac{1}{25}+\frac{1}{20}}} = 0.896$$

Because the calculated value of t = 0.896 is neither less than the lower tail critical value of t = -2.0167, nor greater than the upper tail critical value of t = 2.0167, do not reject the null hypothesis.
Based on these sample data, at the α = 0.05 level of significance there is not sufficient evidence to conclude that the average tensile strength of ropes produced at the two plants is different.

10.55.

a.

H_O: $\pi_1 - \pi_2 = 0$

H_A: $\pi_1 - \pi_2 \neq 0$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{35+35}{50+75} = \frac{70}{125} = 0.56$$

$$z = \frac{p_1 - p_2 - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1-\bar{p})(\frac{1}{n_1}+\frac{1}{n_2})}} = \frac{0.70-0.47-(0)}{\sqrt{0.56(0.44)(\frac{1}{50}+\frac{1}{75})}} = 2.538$$

Since z = 2.538 > 1.96, reject H_O
There is sufficient evidence to conclude that $\pi_1 - \pi_2 \neq 0$.
b. Using the same steps found in part a, indicated in ( ). (1). $\pi_1 - \pi_2$, (2) H_O: $\pi_1 - \pi_2 \geq 0$ vs. H_A: $\pi_1 - \pi_2 < 0$, (3) α = 0.05, (4) the critical value is - 1.645, Reject H_O if z < -1.645, (5) z = 2.538, (6) Since z = 2.538 > -1.645, fail to reject H_O, (7) There is not sufficient evidence to conclude that $\pi_1 - \pi_2 < 0$.

c. Using the same steps found in part a, indicated in ( ). (1). $\pi_1 - \pi_2$, (2) H_O: $\pi_1 - \pi_2 = 0$ vs. H_A: $\pi_1 - \pi_2 > 0$, (3) $\alpha = 0.025$, (4) the critical value is 1.96, Reject H_O if z > 1.96, (5) z = 2.538, (6) Since z = 2.538 > 1.96, reject H_O, (7) There is sufficient evidence to conclude that $\pi_1 - \pi_2 > 0$.

d. Using the same steps found in part a, indicated in ( ). (1). $\pi_1 - \pi_2$, (2) H_O: $\pi_1 - \pi_2 = 0.05$ vs. H_A: $\pi_1 - \pi_2 \neq 0.05$, (3) $\alpha = 0.02$, (4) the critical values are ± 2.33, Reject H_O if z < -2.33 or z > 2.33, (5)

$$z = \frac{p_1 - p_2 - (\text{®}_1 - \text{®}_2)}{\sqrt{\bar{p}(1-\bar{p})(\frac{1}{n_1}+\frac{1}{n_2})}} = \frac{0.70 - 0.47 - (0.05)}{\sqrt{0.56(0.44)(\frac{1}{50}+\frac{1}{75})}} = 1.987$$

(6) Since z = 1.987 < 2.33, fail to reject H_O, (7) There is not sufficient evidence to conclude that $\pi_1 - \pi_2 \neq 0.05$.

10.56.

$$H_o : \text{®}_{2006} \leq \text{®}_{2005}$$

$$H_A : \text{®}_{2006} > \text{®}_{2005}$$

Using an $\alpha = 0.05$, the critical value is z = 1.645. The decision rule based on the z-statistic is: If z calculated > 1.645, reject the null hypothesis; Otherwise, do not reject.

$p_{2005} = \frac{21}{100} = 0.21$ and $p_{2006} = \frac{30}{120} = 0.25$

Using Equation 10-17:

$$\bar{p} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{100(0.21) + 120(0.25)}{100 + 120} = 0.2318$$

Using Equation 10-18

$$z = \frac{(p_1 - p_2) - (\text{®}_1 - \text{®}_2)}{\sqrt{\bar{p}(1-\bar{p})(\frac{1}{n_1}+\frac{1}{n_2})}} = \frac{(0.25 - 0.21) - (0)}{\sqrt{0.2318(1-0.2318)(\frac{1}{100}+\frac{1}{120})}} = 0.7001$$

Since the test statistic, 0.7001, is not greater than the critical value of 1.645, do not reject the null hypothesis and conclude that there is not a higher proportion of U.S. companies offering high-deductible health insurance plans to its workers in 2006 than in 2005.