

Chapter 1

1. A Harris Interactive survey for InterContinental Hotels & Resorts asked respondents, “When traveling internationally, do you generally venture out on your own to experience culture, or stick with your tour group and itineraries?” The survey found that 23% of the respondents stick with their tour group (*USA Today*, January 21, 2004).
 - a. In a sample of six international travelers, what is the probability that two will stick with their tour group?
 - b. In a sample of six international travelers, what is the probability that at least two will stick with their tour group?
 - c. In a sample of 10 international travelers, what is the probability that none will stick with the tour group?

2. Fifty percent of Americans believed the country was in a recession, even though technically the economy had not shown two straight quarters of negative growth (*BusinessWeek*, July 30, 2001). For a sample of 20 Americans, make the following calculations.
 - a. Compute the probability that exactly 12 people believed the country was in a recession.
 - b. Compute the probability that no more than five people believed the country was in a recession.
 - c. How many people would you expect to say the country was in a recession?
 - d. Compute the variance and standard deviation of the number of people who believed the country was in a recession.

3. Twenty-three percent of automobiles are not covered by insurance (CNN, February 23, 2006). On a particular weekend, 35 automobiles are involved in traffic accidents.
 - a. What is the expected number of these automobiles that are not covered by insurance?
 - b. What are the variance and standard deviation?

4. During the period of time that a local university takes phone-in registrations, calls come in at the rate of one every two minutes.
 - a. What is the expected number of calls in one hour?
 - b. What is the probability of three calls in five minutes?
 - c. What is the probability of no calls in a five-minute period?

5. More than 50 million guests stay at bed and breakfasts (B&Bs) each year. The website for the Bed and Breakfast Inns of North America, which averages seven visitors per minute, enables many B&Bs to attract guests (*Time*, September 2001).
 - a. Compute the probability of no website visitors in a one-minute period.
 - b. Compute the probability of two or more website visitors in a one-minute period.
 - c. Compute the probability of one or more website visitors in a 30-second period.
 - d. Compute the probability of five or more website visitors in a one-minute period.

6. Airline passengers arrive randomly and independently at the passenger-screening facility at a major international airport. The mean arrival rate is 10 passengers per minute.
 - a. Compute the probability of no arrivals in a one-minute period.
 - b. Compute the probability that three or fewer passengers arrive in a one-minute period.
 - c. Compute the probability of no arrivals in a 15-second period.
 - d. Compute the probability of at least one arrival in a 15-second period.

7. Axline Computers manufactures personal computers at two plants, one in Texas and the other in Hawaii. The Texas plant has 40 employees; the Hawaii plant has 20. A random sample of 10 employees is to be asked to fill out a benefits questionnaire.

- What is the probability that none of the employees in the sample work at the plant in Hawaii?
- What is the probability that one of the employees in the sample works at the plant in Hawaii?
- What is the probability that two or more of the employees in the sample work at the plant in Hawaii?
- What is the probability that nine of the employees in the sample work at the plant in Texas?

8. The Zagat Restaurant Survey provides food, decor, and service ratings for some of the top restaurants across the United States. For 15 restaurants located in Boston, the average price of a dinner, including one drink and tip, was \$48.60. You are leaving for a business trip to Boston and will eat dinner at three of these restaurants. Your company will reimburse you for a maximum of \$50 per dinner. Business associates familiar with these restaurants have told you that the meal cost at one-third of these restaurants will exceed \$50. Suppose that you randomly select three of these restaurants for dinner.

- What is the probability that none of the meals will exceed the cost covered by your company?
- What is the probability that one of the meals will exceed the cost covered by your company?
- What is the probability that two of the meals will exceed the cost covered by your company?
- What is the probability that all three of the meals will exceed the cost covered by your company?

9. The Troubled Asset Relief Program (TARP), passed by the U.S. Congress in October 2008, provided \$700 billion in assistance for the struggling U.S. economy. Over \$200 billion was given to troubled financial institutions with the hope that there would be an increase in lending to help jump-start the economy. But three months later, a Federal Reserve survey found that two-thirds of the banks that had received TARP funds had tightened terms for business loans (*The Wall Street Journal*, February 3, 2009). Of the ten banks that were the biggest recipients of TARP funds, only three had actually increased lending during this period.

Increased Lending

BB&T
Sun Trust Banks
U.S. Bancorp

Decreased Lending

Bank of America
Capital One
Citigroup
Fifth Third Bancorp
J.P. Morgan Chase
Regions Financial
U.S. Bancorp

For the purposes of this exercise, assume that you will randomly select three of these ten banks for a study that will continue to monitor bank lending practices. Let x be a random variable indicating the number of banks in the study that had increased lending.

- What is $P(0)$? What is your interpretation of this value?
- What is $P(3)$? What is your interpretation of this value?
- Compute $P(1)$ and $P(2)$. Show the probability distribution for the number of banks in the study that had increased lending. What value of x has the highest probability?
- What is the probability that the study will have at least one bank that had increased lending?
- Compute the expected value, variance, and standard deviation for the random variable.

10. The time required to prepare a dry cappuccino using whole milk at the Daily Grind Coffee House is uniformly distributed between 25 and 35 seconds. Assuming a customer has just ordered a whole-milk dry cappuccino,

- a. What is the probability that the preparation time will be more than 29 seconds?
- b. What is the probability that the preparation time will be between 28 and 33 seconds?
- c. What percentage of whole-milk dry cappuccinos will be prepared within 31 seconds?
- d. What is the standard deviation of preparation times for a dry cappuccino using whole milk at the Daily Grind Coffee House?

11. During the busiest time of the day customers arrive at the Daily Grind Coffee House at an average of 15 customers per 20-minute period.

- a. What is the probability that a customer will arrive within the next 3 minutes?
- b. What is the probability that the time between the arrivals of customers is 12 minutes or more?
- c. What is the probability that the next customer will arrive within 4 and 6 minutes?

12. The average amount spent on electronics each year in U.S. households is \$1,250 according to an article in *USA Today* (Michelle Kessler, "Gadget Makers Make Mad Dash to Market," January 4, 2006). Assume that the amount spent on electronics each year has an exponential distribution.

- a. Calculate the probability that a randomly chosen U.S. household would spend more than \$5,000 on electronics.
- b. Compute the probability that a randomly chosen U.S. household would spend more than the average amount spent by U.S. households.
- c. Determine the probability that a randomly chosen U.S. household would spend more than 1 standard deviation below the average amount spent by U.S. households.

$$1. \quad a. \quad P(2) = \binom{6}{2} (.23)^2 (.77)^4 = .2789$$

$$b. \quad P(\text{at least } 2) = 1 - P(0) - P(1)$$

$$= 1 - \binom{6}{0} (.23)^0 (.77)^6 - \binom{6}{1} (.23)^1 (.77)^5$$

$$= 1 - .2084 - .3735 = .4181$$

$$c. \quad P(0) = \binom{10}{0} (.23)^0 (.77)^{10} = .0733$$

$$2. \quad a. \quad P(12) = \frac{20!}{12!8!} (.5)^{12} (.5)^8$$

Using the binomial tables, $P(12) = .1201$

$$b. \quad P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$$

$$.0000 + .0000 + .0002 + .0011 + .0046 + .0148 = .0207$$

$$c. \quad E(x) = np = 20(.5) = 10$$

$$d. \quad \text{Var}(x) = np(1-p) = 20(.5)(.5) = 5$$

$$= \sqrt{5} = 2.24$$

$$3. \quad E(x) = np = 35(.23) = 8.05 \quad (8 \text{ automobiles})$$

$$\text{Var}(x) = np(1-p) = 35(.23)(1-.23) = 6.2$$

$$= \sqrt{6.2} = 2.49$$

$$4. \quad a. \quad 30 \text{ per hour}$$

$$b. \quad \lambda t = 1 (5/2) = 5/2$$

$$P(3) = \frac{(5/2)^3 e^{-(5/2)}}{3!} = .2138$$

$$c. \quad P(0) = \frac{(5/2)^0 e^{-(5/2)}}{0!} = e^{-(5/2)} = .0821$$

$$5. \quad a. \quad P(0) = \frac{7^0 e^{-7}}{0!} = e^{-7} = .0009$$

$$b. \quad \text{probability} = 1 - [P(0) + P(1)]$$

$$P(1) = \frac{7^1 e^{-7}}{1!} = 7e^{-7} = .0064$$

$$\text{probability} = 1 - [.0009 + .0064] = .9927$$

c. }t = 3.5

$$P(0) = \frac{3.5^0 e^{-3.5}}{0!} = e^{-3.5} = .0302$$

$$\text{probability} = 1 - P(0) = 1 - .0302 = .9698$$

d. probability = 1 - [P(0) + P(1) + P(2) + P(3) + P(4)]

$$= 1 - [.0009 + .0064 + .0223 + .0521 + .0912] = .8271$$

6. a. $P(0) = \frac{10^0 e^{-10}}{0!} = e^{-10} = .000045$

b. $P(0) + P(1) + P(2) + P(3)$

$$P(0) = .000045$$

$$P(1) = \frac{10^1 e^{-10}}{1!} = .00045$$

$$\text{Similarly, } P(2) = .00225, P(3) = .0075$$

$$\text{and } P(0) + P(1) + P(2) + P(3) = .010245$$

c. 2.5 arrivals / 15 sec. period Use }t = 2.5

$$P(0) = \frac{2.5^0 e^{-2.5}}{0!} = .0821$$

d. $1 - P(0) = 1 - .0821 = .9179$

7. $N = 60$ $n = 10$

a. $r = 20$ $x = 0$

$$P(0) = \frac{\binom{20}{0} \binom{40}{10}}{\binom{60}{10}} = \frac{(1) \binom{40!}{10!30!}}{\frac{60!}{10!50!}} = \left(\frac{40!}{10!30!} \right) \left(\frac{10!50!}{60!} \right)$$

$$= \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31}{60 \cdot 59 \cdot 58 \cdot 57 \cdot 56 \cdot 55 \cdot 54 \cdot 53 \cdot 52 \cdot 51} \approx .0112$$

b. $r = 20$ $x = 1$

$$P(1) = \frac{\binom{20}{1} \binom{40}{9}}{\binom{60}{10}} = 20 \left(\frac{40!}{9!31!} \right) \left(\frac{10!50!}{60!} \right) \approx .0725$$

- c. $1 - P(0) - P(1) = 1 - .0112 - .0725 = .9163$
 d. Same as the probability one will be from Hawaii; .0725.

$$8. \quad a. \quad P(0) = \frac{\binom{5}{0} \binom{10}{3}}{\binom{15}{3}} = \frac{(1)(120)}{455} = .2637$$

$$b. \quad P(1) = \frac{\binom{5}{1} \binom{10}{2}}{\binom{15}{3}} = \frac{(5)(45)}{455} = .4945$$

$$c. \quad P(2) = \frac{\binom{5}{2} \binom{10}{1}}{\binom{15}{3}} = \frac{(10)(10)}{455} = .2198$$

$$d. \quad P(3) = \frac{\binom{5}{3} \binom{10}{0}}{\binom{15}{3}} = \frac{(10)(1)}{455} = .0220$$

9. Hypergeometric with $N = 10$ and $n = 3$.

- a. $n = 3, x = 0$

$$P(0) = \frac{\binom{3}{0} \binom{7}{3}}{\binom{10}{3}} = \frac{\left(\frac{3!}{0!3!}\right) \left(\frac{7!}{3!4!}\right)}{\frac{10!}{3!7!}} = \frac{(1)(35)}{120} = .2917$$

This is the probability there will be no banks with increased lending in the study.

- b. $n = 3, x = 3$

$$P(3) = \frac{\binom{3}{3} \binom{7}{0}}{\binom{10}{3}} = \frac{\left(\frac{3!}{3!0!}\right) \left(\frac{7!}{0!7!}\right)}{\frac{10!}{3!7!}} = \frac{(1)(1)}{120} = .0083$$

This is the probability there all three banks with increased lending will be in the study. This has a very low probability of happening.

- c. $n = 3, x = 1$

$$P(1) = \frac{\binom{3}{1}\binom{7}{2}}{\binom{10}{3}} = \frac{\left(\frac{3!}{1!2!}\right)\left(\frac{7!}{2!5!}\right)}{\frac{10!}{3!7!}} = \frac{(3)(21)}{120} = .5250$$

$$n = 3, x = 2$$

$$P(2) = \frac{\binom{3}{2}\binom{7}{1}}{\binom{10}{3}} = \frac{\left(\frac{3!}{2!1!}\right)\left(\frac{7!}{1!6!}\right)}{\frac{10!}{3!7!}} = \frac{(3)(7)}{120} = .1750$$

x	$P(x)$
0	0.2917
1	0.5250
2	0.1750
3	0.0083
Total	1.0000

$P(1) = .5250$ has the highest probability showing that there is over a .50 chance that there will be exactly one bank that had increased lending in the study.

d. $P(x \geq 1) = 1 - P(0) = 1 - .2917 = .7083$

There is a reasonably high probability of .7083 that there will be at least one bank that had increased lending in the study.

e. $E(x) = n\left(\frac{X}{N}\right) = 3\left(\frac{3}{10}\right) = .90$

$$\dagger^2 = n\left(\frac{X}{N}\right)\left(1 - \frac{X}{N}\right)\left(\frac{N-n}{N-1}\right) = 3\left(\frac{3}{10}\right)\left(1 - \frac{3}{10}\right)\left(\frac{10-3}{10-1}\right) = .49$$

$$\dagger = \sqrt{\dagger^2} = \sqrt{.49} = .70$$

10. $f(x) = \frac{1}{b-a} = \frac{1}{35-25} = 0.10$

a. $P(x > 29) = 1 - P(x \leq 29) = 1 - .1(29-25) = 0.60$

b. $P(28 \leq x \leq 33) = .1(33-28) = 0.50$

c. $P(x \leq 31) = .1(31-25) = 0.60$ or 60%

d. $= \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(35-25)^2}{12}} = 2.8868$ seconds.

11.

- a. If 15 arrive every 20 minutes on average then $15/20 = 0.75$ arrivals per minute. $P(x \leq 3) = 1 - e^{-\lambda a} = 1 - e^{-0.75(3)} = 0.1054$.
- b. $P(x \geq 12) = e^{-\lambda a} = e^{-0.75(12)} = 0.00012$.
- c. $P(4 \leq x \leq 6) = e^{-0.75(4)} - e^{-0.75(6)} = 0.0498 - 0.0111 = 0.0387$.

12.

- a. The average amount spent $= 1/\lambda = 1,250$. Therefore, $\lambda = 1/1250 = 0.0008$. Since $P(0 \leq x \leq a) = 1 - e^{-\lambda a}$, $P(x > 5,000) = 1 - P(0 \leq x \leq 5,000) = 1 - [1 - e^{-0.0008(5000)}] = 0.0183$
- b. $P(x > 1,250) = 1 - P(0 \leq x \leq 1,250) = 1 - [1 - e^{-0.0008(1250)}] = 0.3679$.
- c. For the exponential distribution the average = standard deviation $= 1/\lambda$. Therefore, one standard deviation below the mean $= 1,250 - 1,250 = 0$. So this exercise is requiring $P(x > 0) = 1$.