An Introduction to Linear Programming

Problem 1 (page 71)

Which of the following mathematical relationships could be found in a linear programming model, and which could not? For the relationships that are unacceptable for linear programs, state why.

$$-1x_1 + 2x_2 - 1x_3 \leq 70$$
$$2x_1 - 2x_3 = 50$$
$$1x_1 - 2x^2_2 + 4x_3 \leq 10$$
$$\sqrt[3]{x_1} + 2x_2 - 1x_3 \geq 15$$
$$1x_1 + 1x_2 + 1x_3 = 6$$
$$2x_1 + 5x_2 + 1x_1x_2 \leq 25$$

Problem 17 (page 74)

GulfGolf also make two different types of baseball gloves: a regular model and a catcher's model. The firm has 900 hours of production time available in its cutting and sewing department, 300 hours available in its finishing department, and 100 hours available in its packaging and shipping department. The production time requirements and the profit contribution per glove are given in the table at the top of the next page.

Model	Production Time (hours)			
	Cutting and Sewing	Finishing	Pack and Ship	Profit/Glove
Regular glove	1	0.5	0.125	$5
Catcher's glove	1.5	0.333	0.25	$8

Assuming that the company is interested in maximizing the total profit contribution, answer the following:

   a. What is the linear programming model for this problem?

   b. Find the optimal solution. How many gloves of each model should GulfGolf manufacture?

   c. What is the total profit contribution the company can earn with the above production quantities?

   d. How many hours of production time will be scheduled in each department?

   e. What is the slack time in each department?

Problem 18 (page 75)

George Johnson recently inherited a large sum of money; he wants to use a portion of this money to set up a trust fund for his two children. The trust fund has two investment options: (1) a bond fund and (2) a stock fund. The projected returns over the life of the investments are 6% for the bond fund and 10% for the stock fund. Whatever portion of the inheritance he finally decides to commit to the trust fund, he wants to invest at least 30% of that amount in the bond fund. In addition, he wants to select a mix that will enable him to obtain a total return of at least 7.5%.

a. Formulate a linear programming model that can be used to determine the percentage that should be allocated to each of the possible investment alternatives.

b. Find the optimal solution using the graphical solution procedure.

Problem 24 (page 76)

Consider the following linear program:

$$\begin{aligned}
\text{Min} \quad & 2x_1 + 2x_2 \\
\text{s.t.} \quad & x_1 + 3x_2 \leq 12 \\
& 3x_1 + x_2 \geq 13 \\
& x_1 - x_2 = 3 \\
& x_1, x_2 \geq 0
\end{aligned}$$

a. Show the feasible region.

b. What are the extreme points of the feasible region?

c. Find the optimal solution using the graphical solution procedure.

Problem 32 (page 79)

Does the following linear program involve infeasibility, unbounded, and/or alternative optimal solutions? Explain.

$$\begin{aligned}
\text{Max} \quad & 4x_1 + 8x_2 \\
\text{s.t.} \quad & 2x_1 + 2x_2 \leq 10 \\
& -x_1 + x_2 \geq 8 \\
& x_1, x_2 \geq 0
\end{aligned}$$

Problem 33 (page 79)

Does the following linear program involve infeasibility, unbounded, and/or alternative optimal solutions? Explain.

$$\begin{aligned}
\text{Max} \quad & x_1 + x_2 \\
\text{s.t.} \quad & 8x_1 + 6x_2 \geq 24 \\
& 4x_1 + 6x_2 \geq -12 \\
& 2x_2 \geq 4 \\
& x_1, x_2 \geq 0
\end{aligned}$$