QM 350

Operations Research
Chapter 4

Problem 1:
A partially completed initial simplex tableau is given:

		x_1	x_2	s_1	s_2	
Basis	c_B	5	9	0	0	
s_1	0	10	9	1	0	**90**
s_2	0	−5	3	0	1	**15**
	z_j					
	$c_j - z_j$					

a. Complete the initial tableau.
b. Which variable would be brought into solution at the first iteration?
c. Write the original linear program.

Problem 2:
The following partial initial simplex tableau is given:

		x_1	x_2	x_3	s_1	s_2	s_3	
Basis	c_B	5	20	25	0	0	0	
		2	1	0	1	0	0	**40**
		0	2	1	0	1	0	**30**
		3	0	−½	0	0	1	**15**
	z_j							
	$c_j - z_j$							

a. Complete the initial tableau.
b. Write the problem in tableau form.
c. What is the initial basis? Does this basis correspond to the origin? Explain.
d. What is the value of the objective function at this initial solution?
e. For the next iteration, which variable should enter the basis, and which variable should leave the basis?
f. How many units of the entering variable will be in the next solution? Before making this first iteration, what do you think will be the value of the objective function after the first iteration?
g. Find the optimal solution using the simplex method.

Problem 3:
Set up the tableau form for the following linear program (do not attempt to solve):

$$
\begin{aligned}
\text{Min} \quad & 4x_1 + 5x_2 + 3x_3 \\
\text{s.t.} \quad & \\
& 4x_1 \qquad\quad + 2x_3 \geq 20 \\
& \qquad 1x_2 - 1x_3 \leq -8 \\
& 1x_1 - 2x_2 \qquad\quad = -5 \\
& 2x_1 + 1x_2 + 1x_3 \leq 12 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Problem 1:

a. Initial Tableau

Basis	c_B	x_1	x_2	s_1	s_2	
		5	9	0	0	
s_1	0	10	9	1	0	90
s_2	0	-5	3	0	1	15
	z_j	0	0	0	0	0
	$c_j - z_j$	5	9	0	0	

b. We would introduce $x2$ at the first iteration.

c. Max $5x1 + 9x2$

s.t. $10x1 + 9x2 \leq 90$

$-5x1 + 3x2 \leq 15$

$x1, x2 \geq 0$

Problem 2 :

Basis	c_B	x_1	x_2	x_3	s_1	s_2	s_3	
		5	20	25	0	0	0	
s_1	0	2	1	0	1	0	0	40
s_2	0	0	2	1	0	1	0	30
s_3	0	3	0	-1/2	0	0	1	15
	z_j	0	0	0	0	0	0	0
	$c_j - z_j$	5	20	25	0	0	0	

b. Max $5x1 + 20x2 + 25x3 + 0s1 + 0s2 + 0s3$

s.t. $2x1 + 1x2 + 1s1 = 40$

$2x2 + 1x3 + 1s2 = 30$

$3x1 - 1/2x3 + 1s3 = 15$

$x1, x2, x3, s1, s2, s3, \geq 0.$

c. The original basis consists of $s1$, $s2$, and $s3$. It is the origin since the nonbasic variables are $x1$, $x2$, and $x3$ and are all zero.

d. 0.

e. $x3$ enters because it has the largest $cj - zj$ and $s2$ will leave because row 2 has the only positive coefficient.

f. 30; objective function value is 30 times 25 or 750.

g. Optimal Solution:

$x1 = 10$ $s1 = 20$

$x2 = 0$ $s2 = 0$

$x3 = 30$ $s3 = 0$

$z = 800.$

Problem 3 :

$$\begin{array}{llllllllllll}
\text{Max} & -4x_1 & - 5x_2 & - 3x_3 & + 0s_1 & + 0s_2 & + 0s_4 & - Ma_1 & - Ma_2 & - Ma_3 & & \\
\text{s.t.} & & & & & & & & & & & \\
 & 4x_1 & & + 2x_3 & - 1s_1 & & & + 1a_1 & & & = & 20 \\
 & & - 1x_2 & + 1x_3 & & - 1s_2 & & & + 1a_2 & & = & 8 \\
 & -1x_1 & + 2x_2 & & & & & & & + 1a_3 & = & 5 \\
 & 2x_1 & + 1x_2 & + 1x_3 & & & + 1s_4 & & & & = & 12
\end{array}$$

$$x_1,\ x_2,\ x_3,\ s_1,\ s_2,\ s_4,\ a_1,\ a_2,\ a_3 \geq 0$$

The problem is infeasible