QM 350

Operations Research

Chapter 4

Problem 1:

Par, Inc., is a small manufacturer of golf equipment and supplies whose management has decided to move into the market for medium- and high-priced golf bags. Par's distributor is enthusiastic about the new product line and has agreed to buy all the golf bags Par produces over the next three months.

After a thorough investigation of the steps involved in manufacturing a golf bag, management determined that each golf bag produced will require the following operations:

1. Cutting and dyeing the material
2. Sewing
3. Finishing (inserting umbrella holder, club separators, etc.)
4. Inspection and packaging

The linear program for this problem is

$$\begin{aligned}
\text{Max}\quad & 10x_1 + 9x_2 \\
\text{s.t.}\quad & \\
& \tfrac{7}{10}x_1 + 1x_2 \le 630 \quad \text{Cutting and dyeing time} \\
& \tfrac{1}{2}x_1 + \tfrac{5}{6}x_2 \le 600 \quad \text{Sewing time} \\
& 1x_1 + \tfrac{2}{3}x_2 \le 708 \quad \text{Finishing time} \\
& \tfrac{1}{10}x_1 + \tfrac{1}{4}x_2 \le 135 \quad \text{Inspection and packaging time} \\
& x_1, x_2 \ge 0
\end{aligned}$$

where

- x_1 : number of standard bags produced
- x_2 : number of deluxe bags produced

The final simplex tableau is

		x_1	x_2	s_1	s_2	s_3	s_4	
Basis	c_B	10	9	0	0	0	0	
x_2	9	0	1	$30/16$	0	$-21/16$	0	**252**
s_2	0	0	0	$-15/16$	1	$5/32$	0	**120**
x_1	10	1	0	$-20/16$	0	$30/16$	0	**540**
s_4	0	0	0	$-11/32$	0	$9/64$	1	**18**
z_j		10	9	$70/16$	0	$111/16$	0	**7668**
$c_j - z_j$		0	0	$-70/16$	0	$-111/16$	0	

a. Calculate the range of optimality for the profit contribution of the standard bag.
b. Calculate the range of optimality for the profit contribution of the deluxe bag.
c. If the profit contribution per deluxe bag drops to \$7 per unit, how will the optimal solution be affected?
d. What unit profit contribution would be necessary for the deluxe bag before Par, Inc., would consider changing its current production plan?
e. If the profit contribution of the deluxe bags can be increased to \$15 per unit, what is the optimal production plan? State what you think will happen before you compute the new optimal solution.
f. Calculate the range of feasibility for b1 (cutting and dyeing capacity).

g. Calculate the range of feasibility for b2 (sewing capacity).
h. Calculate the range of feasibility for b3 (finishing capacity).
i. Calculate the range of feasibility for b4 (inspection and packaging capacity).
j. Which of these four departments would you be interested in scheduling for overtime? Explain.
k. Calculate the final simplex tableau for the Par, Inc., problem (Problem 6) after increasing b1 from 630 to $682\frac{4}{11}$.
l. Would the current basis be optimal if b1 were increased further? If not, what would be the new optimal basis?
m. How much would profit increase if an additional 30 hours became available in the cutting and dyeing department (i.e., if b1 were increased from 630 to 660)?
n. How much would profit decrease if 40 hours were removed from the sewing department?
o. How much would profit decrease if, because of an employee accident, only 570 hours instead of 630 were available in the cutting and dyeing department?
p. Suppose because of some new machinery Par, Inc., was able to make a small reduction in the amount of time it took to do the cutting and dyeing (constraint 1) for a standard bag. What effect would this reduction have on the objective function?
q. Management believes that by buying a new sewing machine, the sewing time for standard bags can be reduced from ½ to ⅓ hour. Do you think this machine would be a good investment? Why?

Problem 2:
Photo Chemicals produces two types of photograph-developing fluids at a cost of $1.00 per gallon. Let Photo Chemicals management requires that at least 30 gallons of product 1 and at least 20 gallons of product 2 be produced. They also require that at least 80 pounds of a perishable raw material be used in production. A linear programming formulation of the problem is as follows:

$$\begin{aligned} \text{Min} \quad & 1x_1 + 1x_2 \\ \text{s.t.} \quad & \\ & 1x_1 \geq 30 && \text{Minimum product 1} \\ & 1x_2 \geq 20 && \text{Minimum product 2} \\ & 1x_1 + 2x_2 \geq 80 && \text{Minimum raw material} \\ & x_1, x_2 \geq 0 \end{aligned}$$

a. Write the dual problem.
b. Use the following optimal simplex tableau for the dual problem to find the optimal production plan for the primal problem:

		u_1	u_2	u_3	s_1	s_2	s_3	
Basis	c_B	15	30	20	0	0	0	
u_1	15	1	0	1	1	0	0	4
u_2	30	0	1	1/4	-1/4	1/2	0	1/2
s_3	0	0	0	3/4	-3/4	-1/2	1	3/2
	z_j	15	30	45/2	15/2	15	0	75
	$c_j - z_j$	0	0	-5/2	-15/2	-15	0	

c. The third constraint involves a management request that the current 80 pounds of a perishable raw material be used. However, after learning that the optimal solution calls for an excess production of five units of product 2, management is reconsidering the raw material requirement. Specifically, you have been asked to identify the cost effect if this constraint is relaxed. Use the dual variable to indicate the change in the cost if only 79 pounds of raw material have to be used.

Problem 1:

a. The final simplex tableau with c1 shown as the coefficient of x_1 is

Basis	c_B	x_1	x_2	s_1	s_2	s_3	s_4	
		c_1	9	0	0	0	0	
x_2	0	0	1	30/16	0	-21/16	0	252
s_2	0	0	0	-15/16	1	5/32	0	120
x_1	c_1	1	0	-20/16	0	30/16	0	540
s_4	0	0	0	-11/32	0	9/64	1	18
	z_j	c_1	9	$(270-20c_1)/16$	0	$(30c_1-189)/16$	0	$2268+540c_1$
	$c_j - z_j$	0	0	$(20c_1-270)/16$	0	$(189-30c_1)/16$	0	

$(20c_1 - 270) / 16 \leq 0 \Rightarrow c_1 \leq 13.5$

$(189 - 30c_1) / 16 \leq 0 \Rightarrow c_1 \geq 6.3$

Range: $6.3 \leq c_1 \leq 13.5$

b. Following a similar procedure for c_2 leads to

$(200 - 30c_2) / 16 \leq 0 \Rightarrow c_2 \geq 6\ 2/3$

$(21c_2 - 300) / 16 \leq 0 \Rightarrow c_2 \leq 14\ 2/7$

Range : $6\ 2/3 \leq c_2 \leq 14\ 2/7$

c. There would be no change in product mix, but profit will drop to 540 (10) + 252 (7) = 7164.

d. It would have to drop below \$6 2/3 or increase above \$14 2/7.

e. We should expect more production of deluxe bags since its profit contribution has increased. The new optimal solution is given by

$x_1 = 300$, $x_2 = 420$

Optimal Value: \$9300

f.

$252 + \Delta b_1 (30/16) \geq 0 \Rightarrow \Delta b_1 \geq -134.4$

$120 + \Delta b_1 (-15/16) \geq 0 \Rightarrow \Delta b_1 \leq 128$

$540 + \Delta b_1 (-20/16) \geq 0 \Rightarrow \Delta b_1 \leq 432$

$18 + \Delta b_1 (-11/32) \geq 0 \Rightarrow \Delta b_1 \leq 52.36$

therefore $-134.4 \leq \Delta b_1 \leq 52.36$

Range: $495.6 \leq b_1 \leq 682.36$

g. $480 \leq b_2$

h. $580 \leq b_3 \leq 900$

i. $117 \leq b_4$

j. The cutting and dyeing and finishing since the dual prices and the allowable increases are positive for both.

k.

Basis	c_B	x_1	x_2	s_1	s_2	s_3	s_4	
		10	9	0	0	0	0	
x_2	9	0	1	30/16	0	-21/16	0	3852/11
s_2	0	0	0	-15/16	1	5/32	0	780/11
x_1	10	1	0	-20/16	0	30/16	0	5220/11
s_4	0	0	0	-11/32	0	9/64	1	0
	z_j	10	9	70/16	0	111/16	0	86,868/11 = $7897\ ^1/_{11}$
	$c_j - z_j$	0	0	-70/16	0	-111/16	0	

l. a. Since this is within the range of feasibility for b_1, the increase in profit is given by

$$\left(\frac{70}{16}\right)30 = \frac{2100}{16}$$

m. It would not decrease since there is already idle time in this department and 600 - 40 = 560 is still within the range of feasibility for b_2.
n. Since 570 is within the range of feasibility for b_1, the lost profit would be equal to

$$\left(\frac{70}{16}\right)60 = \frac{4200}{16}$$

p. The value of the objective function would go up since the first constraint is binding. When there is no idle time, increased efficiency results in increased profits.
q. No. This would just increase the number of idle hours in the sewing department.

Problem 2:

a.

$$\begin{array}{llllllll} \text{Max} & 15u_1 & + & 30u_2 & + & 20u_3 & & \\ \text{s.t.} & & & & & & & \\ & u_1 & & & + & u_3 & \leq & 4 \\ & 0.5u_1 & + & 2u_2 & + & u_3 & \leq & 3 \\ & u_1 & + & u_2 & + & 2u_3 & \leq & 6 \\ & u_1, u_2, u_3 \geq 0 & & & & & & \end{array}$$

b. From the z_j values for the surplus variables we see that the optimal primal solution is $x_1 = 15/2$, $x_2 = 15$, and $x_3 = 0$.
c. The optimal value for the dual is shown in part b to equal 75. Substituting $x_1 = 15/2$ and $x_2 = 15$ into the primal objective function, we find that it gives the same value.

$$4(15/2) + 3(15) = 75$$