CHAPTER 1

PROBLEM 1

Consider a construction company building a 250-unit apartment complex. The project consists of hundreds of activities involving excavating, framing, wiring, plastering, painting, landscaping, and more. Some of the activities must be done sequentially and others can be done simultaneously. Also, some of the activities can be completed faster than normal by purchasing additional resources (workers, equipment, etc.).

a) How could a quantitative approach to decision making be used to solve this problem?
b) What would be the uncontrollable inputs?
c) What would be the decision variables of the mathematical model? the objective function? the constraints?
d) Is the model deterministic or stochastic?
e) Suggest assumptions that could be made to simplify the model.

SOLUTION 1

a) A quantitative approach to decision making can provide a structured way to determine the minimum project completion time based on the activities' normal times and then based on the activities' expedited (reduced) times.
b) Normal and expedited activity completion times; activity expediting costs; funds available for expediting; precedence relationships of the activities.
c) Decision variables--which activities to expedite and by how much, and when to start each activity; objective function--minimize project completion time; constraints--do not violate any activity precedence relationships and do not expedite in excess of the funds available.
d) Stochastic--activity completion times, both normal and expedited, are uncertain and subject to variation; activity expediting costs are uncertain; the number of activities and their precedence relationships might change before the project is completed due to a project design change.
e) Make the model deterministic by assuming normal and expedited activity times are known with certainty and are constant. The same assumption might be made about the other stochastic, uncontrollable inputs.

PROBLEM 2

Consider a department store that must make weekly shipments of a certain product from two different warehouses to four different stores.

a) How could a quantitative approach to decision making be used to solve this problem?
b) What would be the uncontrollable inputs for which data must be gathered?
c) What would be the decision variables of the mathematical model? the objective function? the constraints?
d) Is the model deterministic or stochastic?
e) Suggest assumptions that could be made to simplify the model.

SOLUTION 2

a) A quantitative approach to decision making can provide a systematic way to determine a minimum shipping cost from the warehouses to the stores.
b) Fixed costs and variable shipping costs; the demand each week at each store; the supplies each week at each warehouse.
c) Decision variables--how much to ship from each warehouse to each store; objective function--minimize total shipping costs; constraints--meet the demand at the stores without exceeding the supplies at the warehouses.
d) Stochastic--weekly demands fluctuate as do weekly supplies; transportation costs could vary depending upon the amount shipped, other goods sent with a shipment, etc.
e) Make the model deterministic by assuming fixed shipping costs per item, demand is constant at each store each week, and weekly supplies in the warehouses are constant.

PROBLEM 3

An auctioneer has developed a simple mathematical model for deciding the starting bid he will require when auctioning a used automobile. Essentially, he sets the starting bid at seventy percent of what he predicts the final winning bid will (or should) be. He predicts the winning bid by starting with the car's original selling price and making two deductions, one based on the car's age and the other based on the car's mileage. The age deduction is $800 per year and the mileage deduction is $.025 per mile.

a) Develop the mathematical model that will give the starting bid (B) for a car in terms of the car's original price (P), current age (A) and mileage (M).
b) Suppose a four-year old car with 60,000 miles on the odometer is up for auction. If its original price was $12,500, what starting bid should the auctioneer require?
c) The model is based on what assumptions?

SOLUTION 3

a) The expected winning bid can be expressed as:

$P - 800(A) - .025(M)$

The entire model is:

$B = .7(\text{expected winning bid})$ or

$B = .7(P - 800(A) - .025(M))$ or

$B = .7(P) - 560(A) - .0175(M)$

b) $B = .7(12{,}500) - 560(4) - .0175(60{,}000) = \$5460$.
c) The model assumes that the only factors influencing the value of a used car are the original price, age, and mileage (not condition, rarity, or other factors). Also, it is assumed that age and mileage devalue a car in a linear manner and without limit. (Note, the starting bid for a very old car might be negative.)

PROBLEM 4

A firm manufactures two products made from steel and has just received this month's allocation of b pounds of steel. It takes a_1 pounds of steel to make a unit of product 1 and it takes a_2 pounds of steel to make a unit of product 2. Let x_1 and x_2 denote this month's production level of product 1 and product 2 respectively.

Denote by p_1 and p_2 the unit profits for products 1 and 2, respectively. The manufacturer has a contract calling for at least m units of product 1 this month. The firm's facilities are such that at most u units of product 2 may be produced monthly.

a) Write a mathematical model for this problem.

b) Suppose $b = 2000$, $a_1 = 2$, $a_2 = 3$, $m = 60$, $u = 720$, $p_1 = 100$, $p_2 = 200$. Rewrite the model with these specific values for the uncontrollable inputs.

c) The optimal solution to (b) is $x_1 = 60$ and $x_2 = 626\ 2/3$. If the product were engines, explain why this is not a true optimal solution for the "real-life" problem.

SOLUTION 4

a) The total monthly profit

= (profit per unit of product 1) x (monthly production of product 1)
+ (profit per unit of product 2) x (monthly production of product 2)
= $p_1x_1 + p_2x_2$.

The total amount of steel used during monthly production

= (steel per unit of product 1) x (monthly production of product 1)
+ (steel per unit of product 2) x (monthly production of product 2)
= $a_1x_1 + a_2x_2$.

This quantity must be less than or equal to the allocated b pounds of steel:

$a_1x_1 + a_2x_2 \leq b$.

The monthly production level of product 1 must be greater than or equal to m:

$x_1 \geq m$.

The monthly production level of product 2 must be less than or equal to u:

$x_2 \leq u$.

The production level for product 2 cannot be negative:

$x_2 \geq 0$.

Thus, the model is:

$$\begin{aligned} \text{MAXIMIZE} \quad & p_1x_1 + p_2x_2 \\ \text{s.t.} \quad & a_1x_1 + a_2x_2 \leq b \\ & x_1 \geq m \\ & x_2 \leq u \\ & x_2 \geq 0 \end{aligned}$$

b) Substituting, the model is:

$$\begin{aligned}
\text{MAXIMIZE} \quad & 100x_1 + 200x_2 \\
\text{s.t.} \quad & 2x_1 + 3x_2 \leq 2000 \\
& x_1 \geq 60 \\
& x_2 \leq 720 \\
& x_2 \geq 0
\end{aligned}$$

c) One cannot produce and sell 2/3 of an engine. Thus the problem is further restricted by the fact that both x_1 and x_2 must be integers. They could remain fractions if it is assumed these fractions are work in progress to be completed the next month.