Problem:
Data collected from selected major metropolitan areas in the eastern United States show that 2% of individuals living within the city limits move to the suburbs during a one-year period, while 1% of individuals living in the suburbs move to the city during a one year period. Answer the following questions assuming that this process is modeled by a Markov process with two states: city and suburbs.

a. Prepare the matrix of transition probabilities.

b. Compute the steady-state probabilities.

c. In a particular metropolitan area, 40% of the population lives in the city, and 60% of the population lives in the suburbs. What population changes do your steady-state probabilities project for this metropolitan area?
Answers

a.

\[
\begin{array}{c|cc}
 & \text{City} & \text{Suburbs} \\
\hline
\text{City} & 0.98 & 0.02 \\
\text{Suburbs} & 0.01 & 0.99 \\
\end{array}
\]

b. \[\pi_1 = 0.98 \pi_1 + 0.01 \pi_2 \] (1)

\[\pi_2 = 0.02 \pi_1 + 0.99 \pi_2 \] (2)

\[\pi_1 + \pi_2 = 1 \] (3)

Solving equations (1) and (3) provides

\[0.02 \pi_1 - 0.01 \pi_2 = 0 \]

\[\pi_2 = 1 - \pi_1 \]

Thus,

\[0.02 \pi - 0.01 (1 - \pi_1) = 0 \]

\[0.03 \pi_1 - 0.01 = 0 \]

\[\pi_1 = 0.333 \]

and

\[\pi_2 = 1 - 0.333 = 0.667 \]

c. The area will show increases in the suburb population and decreases in the city population. The current 40% in the city is expected to drop to around 33%.