

**Problem :**

Data collected from selected major metropolitan areas in the eastern United States show that 2% of individuals living within the city limits move to the suburbs during a one-year period, while 1% of individuals living in the suburbs move to the city during a one year period. Answer the following questions assuming that this process is modeled by a Markov process with two states: city and suburbs.

**a.** Prepare the matrix of transition probabilities.

**b.** Compute the steady-state probabilities.

**c.** In a particular metropolitan area, 40% of the population lives in the city, and 60% of the population lives in the suburbs. What population changes do your steady-state probabilities project for this metropolitan area?

## Answers

a.

	City	Suburbs
City	0.98	0.02
Suburbs	0.01	0.99

b.  $\pi_1 = 0.98 \pi_1 + 0.01 \pi_2$  (1)

$$\pi_2 = 0.02 \pi_1 + 0.99 \pi_2 \quad (2)$$

$$\pi_1 + \pi_2 = 1 \quad (3)$$

Solving equations (1) and (3) provides

$$0.02 \pi_1 - 0.01 \pi_2 = 0$$

$$\pi_2 = 1 - \pi_1$$

Thus,  $0.02 \pi - 0.01 (1 - \pi_1) = 0$

$$0.03 \pi_1 - 0.01 = 0$$

$$\pi_1 = 0.333$$

and  $\pi_2 = 1 - 0.333 = 0.667$

- c. The area will show increases in the suburb population and decreases in the city population. The current 40% in the city is expected to drop to around 33%.